Projection.
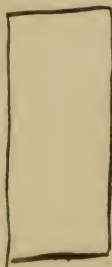
ORTHOGRAPHIC AND ISOMETRICAL

PROJECTION:

DEVELOPMENT OF SURFACES

AND

PENETRATION OF SOLIDS.

BY

ELLIS A. DAVIDSON,

AUTHOR OF "THE ANIMAL KINGDOM EXPLAINED," "DRAWING FOR
CARPENTERS AND JOINERS," ETC.

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CLASSIFIED CONTENTS.

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THE HELIX.

ISOMETRICAL PROJECTION.

INTRODUCTION.

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In the preceding volume,* the construction of figures which possess *length* and *breadth* was taught, all such figures being considered as traced upon one flat surface called a *plane*, thus showing their exact forms as they are really *known to be*.

It now becomes necessary to treat of the delineation of *Solids*—that is, bodies which possess not only length and breadth, but *thickness* as well; and the science by which lines are so disposed that the representation of the object may seem to stand out, or *project* from the flat surface of the paper, is called *Projection*, which is a branch of Solid or Descriptive Geometry.

The subject may be divided into—

Orthographic Projection, by means of which objects are projected by parallel lines from given plans, elevations, or other data, the object being placed in any given position.

Isometrical† *Projection*, by means of which a view of an object is projected at one definite angle, a uniform

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* " Linear Drawing."
† From two Greek words, meaning equal measures.

scale, proportionate to the real measurement, being retained throughout.

Perspective, by which objects are drawn as they *appear* to the eye of the spectator from any point of view that may be selected.

The present volume is devoted to the study of the first and second of these divisions; combining also the mode of obtaining required sections, the methods of describing the peculiar curves generated by one solid body intersecting or *penetrating* another, and the development of surfaces—that is, the construction of the exact shape which a metal plate or other material is to be cut, so as to form or cover the required object in the most ready and accurate manner, and with the least waste—a branch which will be further considered in a subsequent volume, devoted to the technical drawing adapted to the requirements of the metal plate-worker, boiler-maker, and tinman. The lessons are given in as simple a manner as possible, so that the student may be able to follow them with interest, and may be led to desire still further instruction than is here afforded; and it is hoped that the pleasure and benefit he receives from knowledge may awaken in him that spirit of enthusiasm which is the mainspring of all progress. It has been from the *want* of enthusiasm that our workmen have been content with the small amount of knowledge which they have obtained from their "mates" in the shop. It has been this apathy which has caused so many to be satisfied with the "rule

of thumb" instead of the rule of *science.* It is not the province of a work of this kind to dilate on the natural history of enthusiasm ; but our object is to warm up the spirit of our fellow-countrymen—to convince them, that if they will but study the principles of the sciences on which their trades are based, they will, with their acknowledged *manual* superiority, hold their own against the men of every country in the world. Let us, therefore, interpret *interest* in their occupation to imply *enthusiasm;* and let us translate enthusiasm to mean that spirit which urges every man to do his work as well as it can possibly be done, and to develop the mental powers with which his Creator has endowed him to their fullest extent, so that when he leaves the workshop of life he may, in the words of Longfellow, leave "footprints in the sands of time."

ELLIS A. DAVIDSON.

LONDON, *September*, 1868.

A FEW PLAIN HINTS ON

LINEAR DRAWING.

—◆—

LET your paper be rather smaller than your drawing-board, so that the edges may not project.

To fasten the paper down, wet the back, and then paste the edges to the board; let it lie flat whilst drying. This is only necessary when the drawing is likely to be some time in hand; for exercises such as are contained in this volume, it will be sufficient to fasten the paper down by means of *drawing-pins*, which may be bought at one half-penny each.
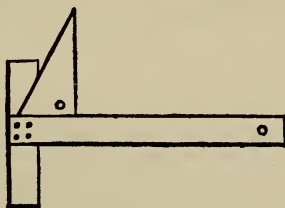
The best T-squares are those where the blade is screwed *over* the butt-end, as in the illustration, as this allows of the "set-square" (or triangle) passing freely along; whilst when the blade is mortised into the butt-end, the set square is stopped when it comes against the projecting edge.

The T-square is to be worked against the left-hand edge of the drawing-board, and should be used for horizontal lines only—perpendiculars are best drawn by working the set-square as above, against the T-square;

for if the T-square be used for perpendicular as well as horizontal lines, the slightest inaccuracy in the truth of the edges of the board would prevent the lines being at right angles to each other.

There are in some cases of mathematical instruments an implement called a "parallel rule," made of two flat pieces of ebony or ivory, connected by two bars of brass. The student is not advised to use these in obtaining parallel lines, as unless the instrument be in very good order, and very carefully used, the lines drawn will not be parallel. The best way to draw lines parallel to each other is by means of two set-squares.* Thus, let it
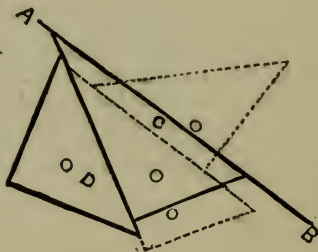
be required to draw several lines parallel to A B. Place the edge of one of your set-squares, C, against the line, and place the other set-square, D, against the first; hold D firmly down, and move C along the edge of D, and thus any number of parallel lines may be drawn; and if lines at right angles to the parallels are required, it is only necessary to hold C, and place D on it, as shown in the dotted portion of the figure.

In inking the drawings, use Indian ink, not writing ink, which rusts the steel of the instruments, and so destroys their refinement. Indian ink may be obtained from two-pence the stick. If you intend inking the drawings, you must work the original pencilling very lightly.

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* Get two set-squares (about sixpence each), the one having angles of 45°, 45°, and 90°, and the other 30°, 60°, 90°.

From the very onset aim at refinement, neatness, and *absolute accuracy*. Do not be satisfied if your work is *nearly* right. Try again, and, if necessary, *again;* and, with increased care and perseverance, success will be the certain result.

MATHEMATICAL INSTRUMENTS

MOSTLY USED.

THE most important instrument is the compass. A complete pair of compasses consists of the body of the instrument, and three movable parts—viz., the steel, the pencil, and the inking-legs which are fixed in their places either by a screw, or by the end of the leg fitting accurately into a socket in the end of the shorter leg of the compass, and kept in its place by a projecting ledge, which runs in a slit in the upper side of the socket.

This is by far the better method, and is used in nearly all modern instruments; its advantages over the screw form are, firstly, that the movable leg only remains firm in its place as long as the thread of the screw is in good order, but the very force used to tighten the pressure, wears the thread away, and then the leg shakes. The consequence of this can be very well imagined, when we remember that one of the leading purposes of compasses is to draw circles, for unless the leg be absolutely firm, the circle will not be true, and the point of the pencil or inking-leg will not meet the starting-point, and so an ugly break will be caused; and secondly, that the screw being but small, is very liable to be lost.

Be careful that in drawing the movable legs out, you do not wrench or bend them from side to side with the view of getting them out more easily, for by that means you will widen the socket, and cause the instrument to

work inaccurately: the proper way is to draw the leg *straight* out.

The steel point is used when distances are to be accurately measured or divided, and therefore compasses which have both points of steel are called "dividers." A pair of these is found in most cases of instruments.

The pencil-leg is used for drawing arcs, circles, &c. Be careful that you keep it exactly the same length as the steel one—this is accomplished by drawing the pencil out a little after each sharpening. In very old-fashioned instruments, the pencil is held in a split tube, which is tightened around it by means of a sliding-ring; but in those of modern make, a short split tube is placed at the end of a solid leg, and the cheeks of this "cannon-leg" are tightened by a screw. This is by far the better construction, as by its means the pencil is not only more firmly held, but the points of the compass may be brought more closely together than in the older form.

The use of the inking-leg (as its name implies) is to repeat the pencil work in ink; the ink must be *Indian* ink, as already mentioned, and it is advisable to mix a small quantity of indigo with it, as otherwise it has a tendency to turn brown. When you mix the Indian ink, do not rub it very hard, as by that means you roughen the edges, and break off small pieces—they may be small indeed (and do we not frequently find failures caused by very trifling obstacles?)—but they work between the nibs of the pen, and cause roughnesses and irregularities of thickness which materially damage a drawing.

On examining the inking-leg, you will find a joint in it, the purpose of this is to enable you to bend the leg at that point, so that the part which contains the ink may be kept perpendicular to the surface of the paper whilst describing a circle, for, if the inking-leg were kept straight as the steel one, when the compass is opened to any extent, only *one* of the nibs (the inner one) will touch the paper, and thus the outer edge of the circle drawn will be ragged and rough. In drawing circles, be careful to lean as lightly as possible on the steel point, so that your centre may not be pricked through the paper, for then, as each concentric circle is drawn, the hole will become larger,

until all chance of following the exact curve will be lost, and when you come to ink the drawing you will find the difficulty still further increased. "Horn centres" are sometimes used. These are small circular pieces of horn with three needle-points fixed in them; one of these may be placed over the centre on the paper, and pressed down; the horn being transparent, the centre-point will be visible through the small plate, and the steel point of the compass may be placed exactly over it. This is all very well in large drawings, and where the circles to be drawn are at some distance from the centre, but where numerous small circles, immediately surrounding the centre, are required, as in the projections of the sections of cones, the horn plate is useless, as it will cover some of the space on which circles are to be drawn; and further, the point resting on it is raised above the surface on which the other is working, and in small circles this will be a disadvantage. The student is therefore reminded of the old adage, "prevention is better than cure," and he is assured that if from the outset he endeavours to *lean* lightly on the instrument, practice will soon place him beyond the necessity for the aid of the horn centre. The following hints will be found useful:—

1. See that the steel point of your compass is *round* and not triangular, which latter form opens the little hole made by far more than if it were round.

2. See that this point is not *too* thin; it should be rather a blunt point than otherwise, only just sharp enough to prevent it slipping away from the centre.

Should either of these two faults exist, they may be easily remedied by drawing the point a few times over an oil-stone, remembering to keep turning it round whilst moving it along.

3. Hold the compass loosely between the *thumb* and *forefinger* only, allowing the instrument to rest with equal weight on both points, and merely using the finger and thumb to support and guide it.

When a circle is required of a larger radius than could be reached with the compass in its usual form, a "lengthening-bar" is used; this is an extra brass rod, which fits into the socket in the leg of the compass, and has at its other end a socket into which the end of the pencil or inking-leg fits. This forms a pair of compasses with one

B

leg very much longer than the other, and which is, there-
fore, rather awkward to manage. Here again the student
is reminded that the pencil-leg and inking-pen must be
bent at the joint, so that they may be perpendicular to
the surface of the paper.

The full-sized compass is, however, not well adapted
for drawing small circles, and, therefore, a complete case
of instruments contains the bow-pencil and the bow-pen.
These are simply small pairs of compasses, the first of
which has a pencil and the other an inking-leg. These
will be found very useful, and may be purchased separately
if not in the case.

For still smaller purposes, " spring-bows " are used ;
these constitute in themselves a small set consisting of
dividers, pencil, and inking-bows. The legs instead of
being united by a hinge-joint, are made in one piece, so
as to form a spring, which by its action tends to force the
points apart ; they are then acted upon by a nut, which,
screwing upon a bar fixed in one leg and passing through
the other, closes the legs in the most minute degree pos-
sible. These will be found of immense service in the
higher branches of mechanical and architectural drawing
where very small arcs and circles are required, as in the
delineation of the teeth of wheels, mouldings, and other
architectural details.

Another important instrument is the drawing-pen,
which is something like the inking-leg of the compass
already described; it is, however, generally smaller in its
nibs, and is fitted on to an ivory or ebony handle. The
ink should be placed between the nibs by means of a
camel's hair brush. The pen should be held *nearly up-
right*, with its flatter side next to the rule, the end of the
middle finger resting on the head of the screw. Before
you ink any line of your drawing, be careful to try your
pen on another piece of paper, in order that you may
ascertain whether the line drawn by the pen would be
of the proper thickness, and if not, the pen may be ad-
justed by means of the screw, which acts in a way similar
to the screw on the spring - bows already described.
Before putting your inking-leg or drawing-pen away, be
sure to wipe it well, and finally to pass a piece of paper
between the nibs, so as to remove any ink that may have
dried, or any grit which may have been deposited.

The rule, or straight-edge, which you use when inking your lines, must have a bevelled edge ; and further, the bevel must be turned *downwards towards the paper;* this will avoid any smearing which might occur if the edge of the rule were to touch the paper whilst the line is wet.

Scales of different sorts are used in mechanical and architectural drawing ; but as the subject of the present volume does not necessarily involve working "to scale," the uses and construction of these will be found appended to the volumes on the above-named sections of scientific drawing.

The protractor (the brass semicircle used in measuring and constructing angles) has been described, and its uses fully explained in connection with Fig. 44, page 31, of the volume on "Linear Drawing ;" repetition here is therefore unnecessary, and we proceed to mention what are called "French curves." These are rules cut into an almost endless variety of shapes, one of which is here shown : they are used in inking curves. To do this,

you must turn your French curve about, until some part of it corresponds with the form already drawn in pencil, which may then be repeated in ink, the pen being guided by the French curve. If you cannot find any portion of your rule which will correspond with the whole of your pencilled curve, draw as much of it as you can, and then find the remainder at some other part of your French curve, or on another one. As these useful implements may be had in innumerable patterns, from five shillings per dozen, the student is advised to provide himself with two or three of them ; but the author wishes it to be plainly understood that he does not imply that by means of French curves freehand drawing may be dispensed with. On the contrary, he urges this practice on

all students; for there is such variety of form in drawing that no mechanical means can possibly supersede the necessity for the accurate and refined education of the eye which is obtained by that study; and further, a little practice will enable students to draw many curves by hand in less time than it would take them to find their places on the French curve.

It is not within the province of this work to recommend the instruments of any particular maker, nor to suggest the prices which should be given, as this last depends on the means at the command of the purchaser. The *cheapest* sets of instruments are the French or Swiss, which for the price (from 1s. 6d., upwards) are quite as much as could be expected; but smallness of price is not always real cheapness, and a good article, manufactured by, and bearing the name of, a respectable English house, will be found by far the most economical in the end.

As to pencils, the degrees most generally useful are those marked HB and H, the latter of which being harder than the former, is more adapted for very minute work; but, as a rule, hard pencils are not the best for mechanical drawings which are to be inked, as they are liable to make grooves in the paper, the bottom of which the nib of the drawing-pen does not touch, and hence the edges of the line will be ragged; and further, lines which are drawn with very hard pencils are difficult to rub out. For mechanical drawing, it is best to cut a *flat* point to the pencil; this is done by cutting away the wood, and leaving about an eighth of an inch of lead projecting, which is then to be cut until it is thinned to a flat, broad point like a chisel; the broad side of this point is moved along against the rule, and the line thus drawn will be found to be much finer than one drawn with a round point. The chisel-point is economical in various ways, for it will not break so often, and the point once cut can be rubbed from time to time on a piece of fine glass-paper or file, or even on the edge of the drawing-paper. Much of the time which would be otherwise spent in sharpening the points is thus saved, and the expense of numerous pencils is at the same time diminished.

Once again the student is urged to remember that the mere possession of a case of instruments, however good.

will not constitute a draughtsman. The instruments are merely the tools—the mechanical agents through which the mind acts ; and it cannot be denied that the more the mind comprehends of the subject to be drawn, the more willing and intelligent servants will the hands become, and the more accurately will they guide the compass or the drawing-pen. Geometrical drawing, then, should be looked upon as a mental exercise more than a merely manual occupation or employment, giving us not only subject for thought and earnest reflection, but enabling us to communicate our plans to others in such a manner that they can understand us and work out our designs better than they could have done from the most eloquent description.

The student will, no doubt, find it difficult at first to draw very fine lines, or to get them to intersect each other exactly as required, especially if he has been engaged in some hard manual occupation during the day ; but he will find a little practice will soon overcome this, if he but starts with patience, energy, and the earnest desire to excel.

ELEMENTARY PRINCIPLES,

ILLUSTRATED BY

Plate I.

Fig. 1.—If we place two planes or surfaces at right angles to each other, so as to form a floor and a wall, the floor, A B, is called the horizontal, and the wall, C D, the vertical, planes of projection.

The Projection of Lines.

No. 1.—Let us take a piece of wire, and fix it in an upright position, *a b*, then the point on which the wire rests is called the horizontal projection, or *plan ;* and if we carry lines directly back from its extremities until

FIG. 1.

FIG. 2.

Plate I.

they cut the vertical plane in *c* and *d*, the line *c d* is the vertical projection, or *elevation*, of the wire.

No. 2.—If a wire, *e f*, be fixed at right angles to the vertical plane, the point *f*, in which it is fixed, is the *elevation*, being the view which would be obtained if the model were placed on an exact level with the eye, the point *e* being immediately opposite the spectator, so that the end only of the wire could be seen. If now, perpendiculars are dropped from *e* and *f* until they meet the horizontal plane in *g* and *h*, the line uniting *g* and *h* will be the *plan* of the wire, or the view obtained by looking straight down on it.*

Further, if we suppose a wire, *i j*, No. 3, to be suspended in space, perpendiculars dropped from its extremities to cut the horizontal plane, will give the *plan k l;* then, if lines be drawn from *k* and *l* to meet the vertical plane in *m* and *n*, and perpendiculars be raised from these points, intersected by lines drawn from the ends of the wire parallel to *k m* and *l n*, the points *o* and *p* will be obtained, and the line joining these will be the *elevation* of the wire *i j*.

In the model used for illustrating this lesson, the vertical and horizontal planes are connected by hinges, and are kept at right angles to each other by means of a brass loop. If, now, the wires be removed, and the pin, *r*, be withdrawn, so as to allow the plane, C D, to fall backward, the two planes of projection will form one surface, separated only by the line I L (Fig. 2), and the plans and elevations will be seen in the positions in which they are placed in projection.

The line separating the two planes is called the *intersecting line*, and will be lettered I L throughout this book.

It must be borne in mind that the "*plan*" of an object does not mean merely the piece of ground *it stands* upon, but the space it overhangs as well: thus, the piece of ground on which the small lodge, Plate II., No. 1, would stand, is represented by the dotted square in the plan, whilst the true space which the building covers or overhangs is represented by the outer square.

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* It must be remembered that in projection the visual rays are supposed to be *parallel* to each other, and not *convergent*,† as in pictorial perspective.
† *Convergent.* From con, *with*, and vergo, *to incline* (Latin). Arising in various points and approaching each other until they meet.

It will be seen that in all the figures shown in Plate I. the lengths of the plans and the heights of the elevations are the same as the heights and lengths of the objects they represent ; thus, *c d* is the same length as *a b*, and *k l* and *o p* are the same length as *i j ;* but plans are not always the size, nor are elevations always the full height, of the object, both being dependent on the position or angle in which the subject to be drawn is placed. Before proceeding to treat of the changes which lines undergo by alteration of position, it is necessary that the terms used to define such positions should be understood, and for this purpose we again refer to Plate I.

Here we have the line *a b* standing upright on the floor of the model, and as its distance from the wall is the same throughout its entire length, it is said to be *at right angles (or perpendicular) to the horizontal, and parallel to the vertical plane*. In No. 2 the line *e f* is said to be at *right angles to the vertical, and parallel to the horizontal plane;* and it is evident that the line *i j*, No. 3, is *parallel to both planes*.

It will be seen that whilst the plan of a line when standing upright is a mere point (Plate I., Fig. 2, *a*), the plan of the same line, when placed horizontally, as *k l*, is the full length of the original. The figures in the next plate will account for this difference, and will show how the length of the plan is dependent on the angle at which the line is inclined.

Plate II.

Let the original position of a wire (Fig. 2) be perfectly upright, then its plan will be the point *a*, and its elevation the line *b c*.

Now, if this wire be made to work on a hinge-joint at *b*, and if the end *c* be moved from left to right, as from *c* to *d*, the end, *d*, being kept the same distance from the wall of the model, the wire will still be "*parallel* to the vertical, but *inclined* to the *horizontal* plane (of course it may be inclined at any angle ; in this case it is at 60°).

To find the plan of this wire, draw a line from *a*, parallel to I L. From *d* drop a perpendicular to cut this line ; then *a e* is the plan of *b d* in the position in

Plate II.

which it is now placed (viz., parallel to the vertical, and inclined at 60° to the horizontal plane) : and if the movement of the wire were continued until it reached *f* (it would then be parallel to both planes), the plan would be the same line extended to *g*.

The line *b d* is said to be placed at a simple angle, because it is inclined to *one* plane, but remains parallel to the other.   Let us now suppose the wire fixed in this slanting position, as far as its inclination to the horizontal plane is concerned—but if the whole hinge is made to rotate on a pivot, so that, without altering the slant, the end, *d*, may be turned forward—the line will then be at a *compound* angle, that is, it will be inclined, or slanting, to *both* planes.

Now, it will be remembered that, although we have turned the wire round, we have not altered its slant to the horizontal plane ; it will therefore overhang a piece of ground of exactly the same shape and size as it did in Fig. 2 ; but the position of that space will be changed. Let us now assume that in addition to the wire being inclined at 60° to the horizontal, it is required to slant at 45° to the vertical plane.   Place the plan, *h i*, Fig. 3, at 45° to the intersecting line, and draw perpendiculars from its extremities, the line from *h* will cut the intersecting line in *j*, and will give the base of the line.   To find its height, we must remember that, although we turned the wire round, we did not alter its slant, and therefore the height of the end, *d*, remains the same as it was ; so that a horizontal line being drawn from *d*, Fig. 2, to meet the perpendicular drawn from *i*, in the point *k*, the line *j k* will be the *projection* of the wire inclined to both planes at the required angles.

It will be seen that in this case both plan and elevation are shorter than the line itself.

Exercise.—To find the real length of a line when it is inclined to both planes, and its plan, *h i*, and the height of the end is given.   Draw a line, *i l*, at right angles to *h i*, and make it equal to the given height.   *Then the line h l will be the real length ;* for the plan, the original line, and a perpendicular dropped from its end, form a right-angled triangle; and this triangle, instead of standing upright, as in Fig. 2, is placed horizontally in Fig. 3 ; and the line *h l* will thus be found to be of the same length as *b d*.   This

may be illustrated by holding a set-square vertically, and rotating it on its edge until it lies on the horizontal plane; the real length of the long edge (or hypothenuse), which when the set-square was vertical was represented by its plan, *h i*, will then become visible.

Fig. 4.—Again, it has been shown that if a wire were fixed at *o*, at right angles to the vertical, and parallel to the horizontal plane, its plan would be *m n*, and its elevation the point *o;* and if it were rotated on the point *n* until it became parallel to I L, its plan would be *n p* and its elevation *o q;* but, on the principle shown in Figs. 2 and 3, it will be evident that if the wire be rotated only as far as *r*, the elevation of it will be the line *o s*.

Plate III.—The Projection of Planes or Surfaces.

The same laws which guide the projection of single lines will also govern the delineation of planes, which are flat surfaces bounded by lines. Let *a b c d*, Fig. 1, be a metal plate, the surface of which is parallel to the vertical and perpendicular to the horizontal plane: its *plan* will then be the line *a' b'*. If now this plane be turned, so as to be at right angles to both planes, its plan—that is, the line on which it would stand—will be *a' b'*, Fig. 2, and its elevation the line *a'' c''*, or the view obtained when looking straight at the long edge.

Now, let this plane rotate on the line *a'' c''*, as a door on its hinges, until the plan reaches *b''*, then a perpendicular drawn from *b'* will give the rectangle *a'' c'' b''' d'''*, which will be the projection of the plane, when perpendicular to the horizontal and inclined to the vertical plane, the height remaining unaltered. The other rectangles show the projections of the plane when further rotated.

Fig. 3.—In this figure the plane again rests on *a b*, its edge, *b d*, only being visible in the elevation; but this edge hides the opposite one, which is parallel to it, and therefore the points *a* and *c''* are immediately at the back of, or "beyond," *b d*. Let us now rotate the plane on *a b*, as in closing a box-lid or trap-door, then the plan of the plane will be the rectangle *a b c'' d'';* and the more the plane is lowered, the longer the plan will become, as is shown at *e* and *f*. Notwithstanding the slanting direction which the plane has asumed in relation to the horizontal plane, it
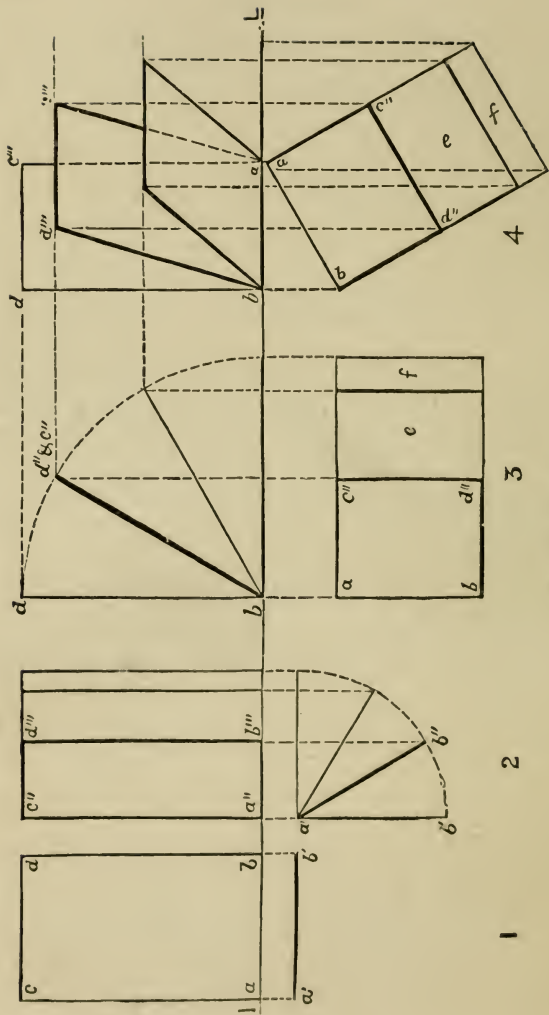
Plate III.

still remains at right angles to the vertical plane. This is shown in the plan, where the lines *a b* and *c'' d''*, which represent the upper and lower edge of the plane, are perpendicular to I L. Let us now place the plane at a compound angle—this will be done by rotating the plan (*carefully lettered*, as in Fig. 3)—then, perpendiculars drawn from each of the points, intersected by horizontal lines from the corresponding points in the elevation, will give the required projection. The process is so plainly shown in the illustration that it is believed further explanation will be unnecessary.

The student is urgently recommended not to be content with simply copying the diagrams herein given, which are merely to be considered as illustrations of principles ; and thus, unless those principles be understood and applied, nothing will be gained. He is therefore advised to vary the form of the plane, and to project it at various angles.

Plates IV. and V. are familiar applications of the foregoing lessons.

Plate IV.

Represents a door when the wall is parallel to the vertical plane, the door being at an angle to it.

The plan should be drawn firstly, and the elevation projected from it.

Plate V.

Fig. 1 represents a trap-door and framing, the door being inclined to the horizontal plane, supported in that position by a piece of timber. In this figure the plan of the framing should be drawn firstly ; then its elevation. To this elevation the edge of the trap-door should be added, which should then be projected on to the plan.

In Fig. 2, the entire plan rotated should be drawn firstly, and the projection obtained by drawing perpendiculars from the angles, and intersecting them by horizontals drawn from the corresponding points in the elevation.

Plate VI.

Fig. 1.—Here a plane square is placed with its surface parallel to the horizontal plane, and its edges, *a b c d*,
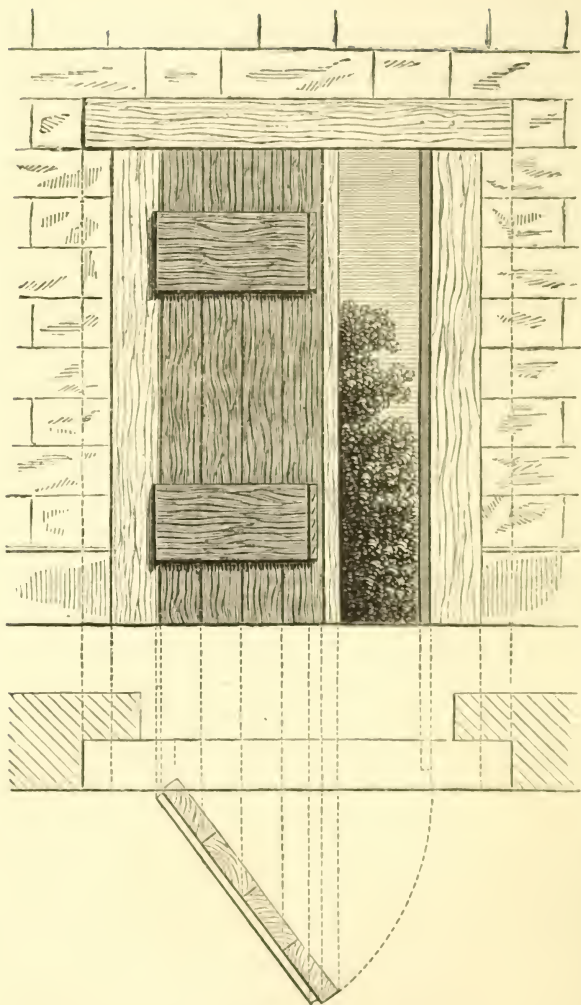
Plate IV.
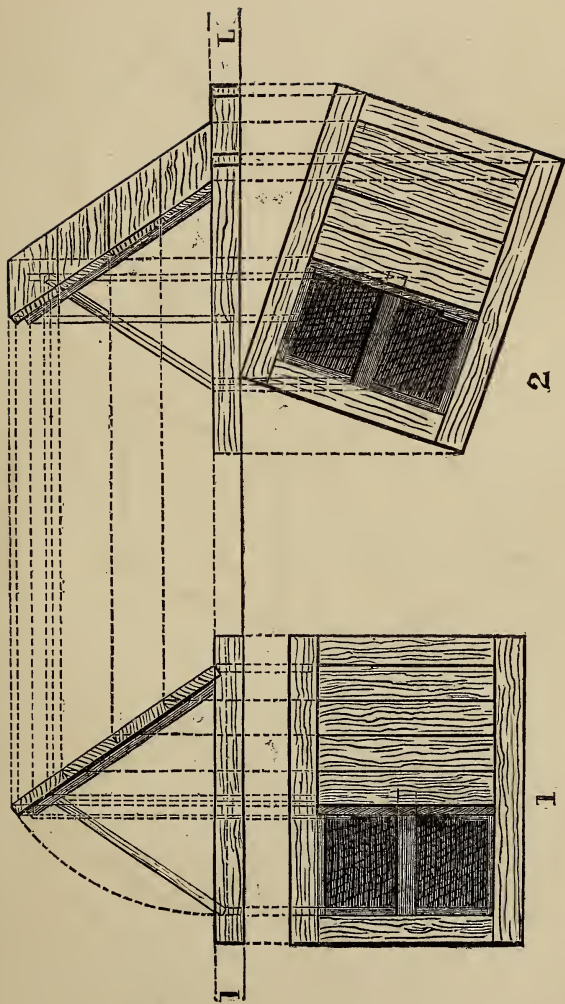
Plate V.
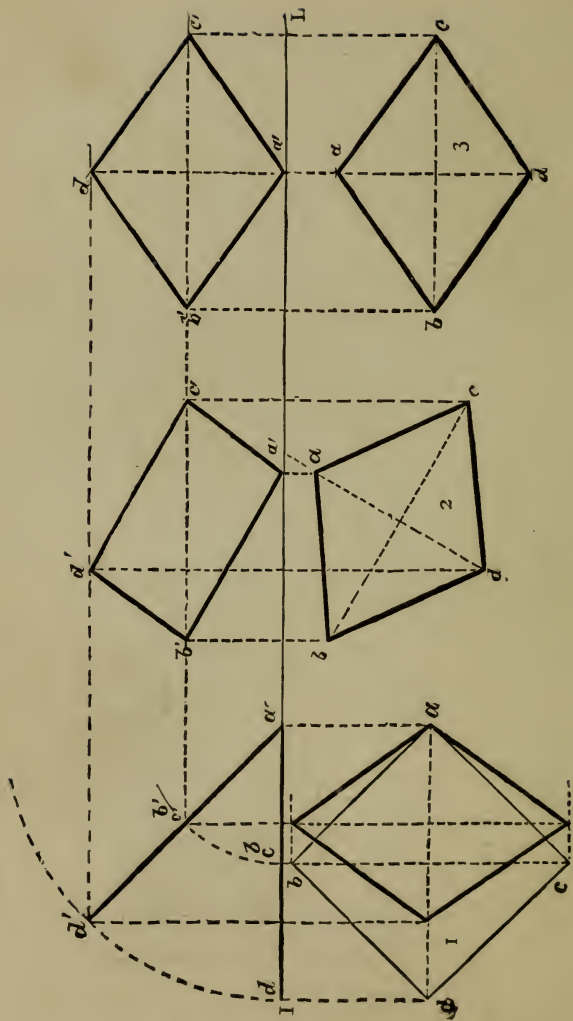
Plate VI.

making angles of 45° with the vertical plane. As this plane is supposed to possess little or no thickness, its elevation, when lying flat, is merely the line $a'\,d'$; the angle c, and d which lies directly beyond it, being marked ^b_c. If we now raise the square, allowing it to rest on the angle a, the extremities of the diagonals, d, c and b, will travel through parts of circles. Thus, let it be required that the diagonal $a\,d$ shall be parallel to the vertical plane, and inclined to the horizontal at 45°. Draw a perpendicular from a to the intersecting line, and thus obtain a'. From a' draw a line at 45°, and with radius $a'\,c$ and $a'\,d$ describe arcs cutting the inclined line in $^{b'}_{c'}$, and d'; the extremities of the diagonals are thus transferred from the horizontal to the inclined elevation.

Now, the points ^b_c and d, in rising higher, will also have moved towards a in the plan, in the track indicated by dotted lines; their present position is determined by dropping perpendiculars from $^{b'}_{c'}$ and d' to cut the dotted lines, and the points being united by lines, the plan of the square in the required position will be obtained.

Let it now be required to obtain the projection of this square, when, in addition to the diagonal $a\,d$ being inclined at 45° to the horizontal, it is inclined at 60° to the vertical plane; in other words, keeping the square resting on the point a', inclined at its present angle, and rotating it. The plan then will be the same as in Fig. 1, but turned round until $a'\,d'$ is at 60° to the intersecting line; then perpendiculars raised from each of the angles, intersected by horizontals from the corresponding points in the previous elevation, will give the projection in Fig. 2.

The same plan turned so that $a\,d$ is at right angles to the intersecting line, and worked out as in the last figure, will give the projection of the square when resting on one of its angles, its plane being at 45° to both the planes of projection. It will be seen that the diagonal, $c\,b$, has, in all three figures, remained parallel to the *horizontal* plane; but in Fig. 3 it will be observed to be parallel to *both* planes.

The student who has thoroughly mastered the foregoing lessons will have seen that, when he understood the projection of *single lines*, he soon comprehended the delineation of planes, since planes are but forms

C

bounded by lines. It is hoped that the next step, the pro-
jection of solids,* may be divested of some of its appa-
rent difficulties, by the reflection that solids are made up
of planes,† and that thus, when planes can be projected
separately, it will be easy to work out several combined
in one object. Thus a cube, or solid square, is formed of
six equal squares ; and as each of these sides is parallel
to the opposite one, the trouble will not be much more
than projecting three planes.

Cubes and Prisms.

When three or more planes meet at one point, as at the
corners of a cube, they form a *solid angle*.

A prism is a solid whose opposite ends are equal and
similar plane figures, and whose sides, uniting the ends,
are parallelograms.

The ends of prisms may be either triangles, squares, or
polygons.

A line drawn from the centre of one end of a prism to
the centre of the other is called the *axis*.

Plate VII.—To Project a Cube.

First position, Fig. 1.—When standing on the horizontal
plane, its axis being vertical, and its sides at 45° to the
vertical plane.

Let *a b c d* be the plan of the cube, and *e* the plan of
the axis. Draw perpendiculars from each of the angles of
the plan, and make the height above the intersecting line
equal to the side of the plan. Draw the top line, *a d*,
which will complete the elevation, the axis being hidden
by the edge, *c*.

Second position, Fig. 2.—When resting on the solid
angle, *a*, its axis being inclined at 65° to the horizontal,
and parallel to the vertical plane.

As the axis of a prism is *parallel* to *its edges*, it will
only be necessary to place the elevation of Fig. 1 so that
the edges are at 65°, then the axis will be between the
edge *c c* in the front, and *b b* beyond ; and as the diagonal,
a d, which forms the breadth of the base, is at right
angles (90°) to the edge, *a a*, the plane of the base will be

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* From the Latin solidus, *compact*.
† Excepting the sphere and its allied forms, no portions of which are
absolute planes.

Plate VII.

C 2
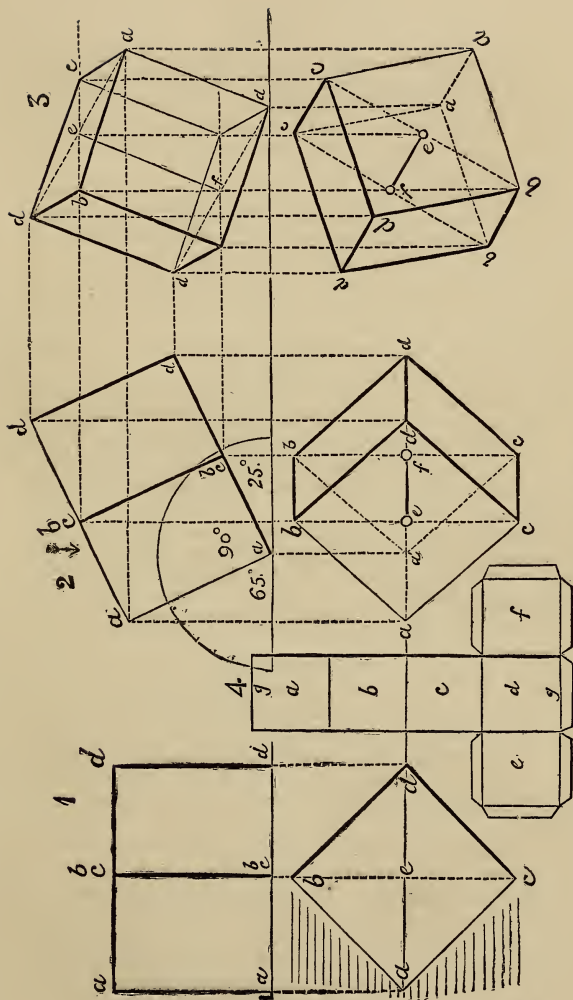
at 25° to the horizontal plane. Perpendiculars dropped
from the angles of this elevation, intersected by horizontals
drawn from the corresponding points in the plan of Fig. 1,
will give the plan of Fig. 2, or the view obtained by looking
down on the elevation, in the direction of the arrow. The
axis, *e f,* will now be seen.

The student is urged to letter with the utmost care until
he has become accustomed to follow each point through
its various changes of position. In this plate, and all
subsequent projections of prisms, the points of the
base, or lower end, will be marked with the same letters
as those of the opposite, or upper end, but in smaller
characters.

Fig 3.—When the axis of the cube is at 65° to the
horizontal and 30° to the vertical plane.

Place the plan so that the line of the axis, *e f,* is at 30°
to the intersecting line. Draw perpendiculars from the
solid angles of the plan, and horizontals from the corre-
sponding points in the elevation of Fig. 2. The intersect-
ing of these two sets of lines will give the points for the
projections.

Shade Lines.—The light has been supposed to come
in the direction of the parallel lines on the left of the
point *a.* Thus, the sides *c d* and *d b* are in shade. This
is indicated by the lines on the plan being darker than
the others, and all perpendiculars rising from them will
be dark also.

Development.—The development is formed by the
shapes of all the sides of an object being laid down on a
flat surface, so that when folded, or connected, a given
solid may be either constructed or covered. By "solid"
is here meant an object that has the external appear-
ance of solidity. Whether the body be really solid or
hollow will be subsequently determined by sections or
cuttings.

To Develop a Cube (Fig. 4.)—A cube consists of six
square sides. Let *a b c d* be four of these, which, uniting at
g g, will form the walls, then *e* and *f* will be the top and
bottom. A very useful model may be thus formed. The
strips left at the edges will be found useful in fastening
the sides together. If the model is made of cardboard
the lines should be cut half through, and half the thick-
ness of the strips peeled off.

Plate VIII.

To Project a Square Prism.— Width of side $\frac{1}{2}$ inch, length $1\frac{1}{2}$ (or 1·5) inch.

The prism is in this lesson placed so that its axis is vertical, and its long faces are at 45° to the vertical plane.

Draw the square, Fig. 1, which is the plan of the prism, its sides being at 45° to the intersecting line ; and perpendiculars drawn from the angles will give the edges of the elevation, which are to be terminated by a horizontal at $1\frac{1}{2}$ inch from the base.

Fig. 2.—It is now required to draw the elevation and plan when the axis, although remaining parallel to the vertical, is at 35° to the horizontal plane.

Now it will be evident, that as far as the elevation is concerned, it will merely be altered in *position*, not in *form*, which change is effected by allowing the object to rest on one angle of the base, and continuing the motion until one edge of the elevation (the edges being parallel to the axis) is at 35° to the horizontal plane. It will therefore only be necessary to copy the previous elevation, inclining it at the required angle. This motion, however, whilst causing so slight an alteration in the elevation, causes an entire change in the plan ; for whilst in the first position the plans of the edges were mere *points* (see Fig. 1, Plate I.), which united form the base, the square of the top being immediately *over* this, as in a line placed vertically, the upper extremity is directly over the lower ; but the moment the line is inclined, the plan, which was previously a point, becomes a line, the length of which increases as the object approaches the horizontal. (See Fig. 2, Plate II.)

But, although the position of the lines is altered, as far as their relation to the horizontal plane is concerned, they still remain *parallel* to the *vertical* plane ; and if the eye were placed immediately over the object, the widths across from the front to the back would be seen to be the same throughout the motion. Therefore, from the angles of the plan of Fig. 1 draw horizontal lines, which will give the widths of the two upper sides ; the two under them being the same, will be hidden by them. The length of the diagonal of the top and
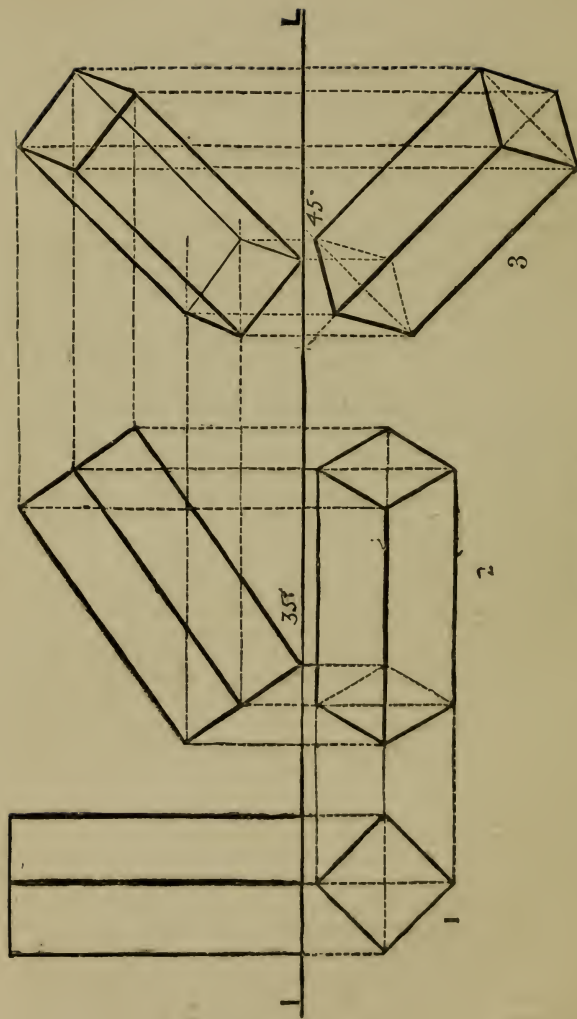
Plate VIII.

bottom, which is at right angles to the vertical plane, thus remains unaltered, but the diagonal which is inclined will necessarily become shortened. This will be seen in continuing the projection of the plan. Draw perpendiculars from the two extremities of the line which is the edge elevation of the end, to cut the middle line of the three horizontals previously drawn in the lower plane. From the middle point of the edge elevation then draw a perpendicular which will cut the *two* outer horizontal lines, and thus four points will be obtained, and these united will give the *lozenge*, which is the plan of the *square* end when inclined. (Refer to Fig. 1, Plate VI.) The lower end of the prism will be obtained in a similar manner.

Fig. 3.—It is now required that the object shall be rotated on its solid angle, so that the axis shall be at a *compound* angle—that is, it shall not only be obliquely placed in relation to the horizontal, but to the vertical plane. This operation has been shown in Fig. 3, Plate II., and it is therefore only necessary to remind the student that, so long as the inclination of a line in relation to the horizontal plane is not altered, no change but that of *position* will occur in the plan; for, however much the object may be rotated *horizontally*, the length of the space it overhangs will not be extended, nor will the *heights* of any of the points be altered; and this knowledge is the key to the projection of Fig. 3.

Place the plan (of Fig. 2) so that its axis and the edges parallel to it are at 45° to the intersecting line, then from each point in the plan raise perpendiculars, and intersect them by horizontals drawn from the corresponding angles in the elevation of Fig. 2. Join the points so obtained, and the result will be the projection shown in the upper figure of No. 3.

Plate IX.—Sections.

Fig. 1 is the plan and elevation of the square prism forming the subject of the last exercise. It is required to find the true shape of a section or cutting, caused by a plane passing through the prism in the direction of the line *a d*. This plane of section would cut through the diagonal *a c* of the top and the angle *d* of the bottom. Draw the dotted lines *a c* and *d′ d′* at right angles to the
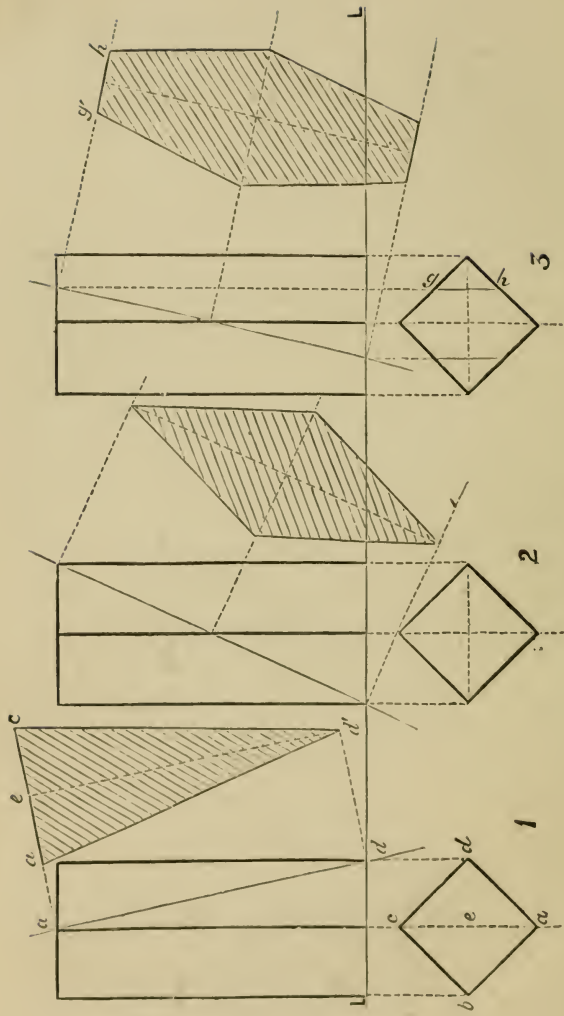
Plate IX.

line of section, and at any part draw *d′ e* parallel to *a d*. Now, it will be evident that this will be the greatest length of the section, and that the *width* will be *somewhere* on each side of *e ;* but where? How *wide will* the section be? These are questions which the student will do well to ask himself.

Now it is clear that in passing through *a c*, the section-line cuts the object in the widest part ; therefore, if the eye be carried down from *a* in the elevation to *a c* in the plan, it will be seen that the real width on each side of the centre *e* is *e a* and *e c ;* therefore, if these lengths be set off on each side of *e* in the section-line, and the points joined to *d′*, then *a c d′* will be the true section.

In Fig. 2, the section-plane passes from one angle of the top to the opposite angle of the bottom, cutting through the middle of the two edges. The length will of course be equal to that of the section-line and the width, *a c*, as in the last figure.

In Fig. 3, the section-plane passes from a line connecting the middle points of two adjacent edges of the top, to a similar line on the two opposite edges of the base. The width of the section at its middle will be equal to the diagonal *a c*, and at the top and bottom it will be equal to *g h*.

It is usual to cover sections with lines at 45° to their central line.

Plate X.

Fig. 1 is the plan and elevation of a square prism, similar to that which formed the subject of the last lesson. Now, if this be made of wood, and cut so that the section passes through the axis at 45°, and a pin, *c*, be fixed in the centre of the section, at right angles to its surface, the upper portion may be rotated on the pin, so that the short line (*a*) will move to *b*, and be at right angles to it, and the object will be represented by the elevation and plan in Fig. 2.

Fig. 3 is the development, which will show how a metal plate may be cut without waste, so as to make a square pipe to turn a corner, or form an elbow. The true section is shown in Fig. 4, its length being equal to *d e*, and its width to *f g*.
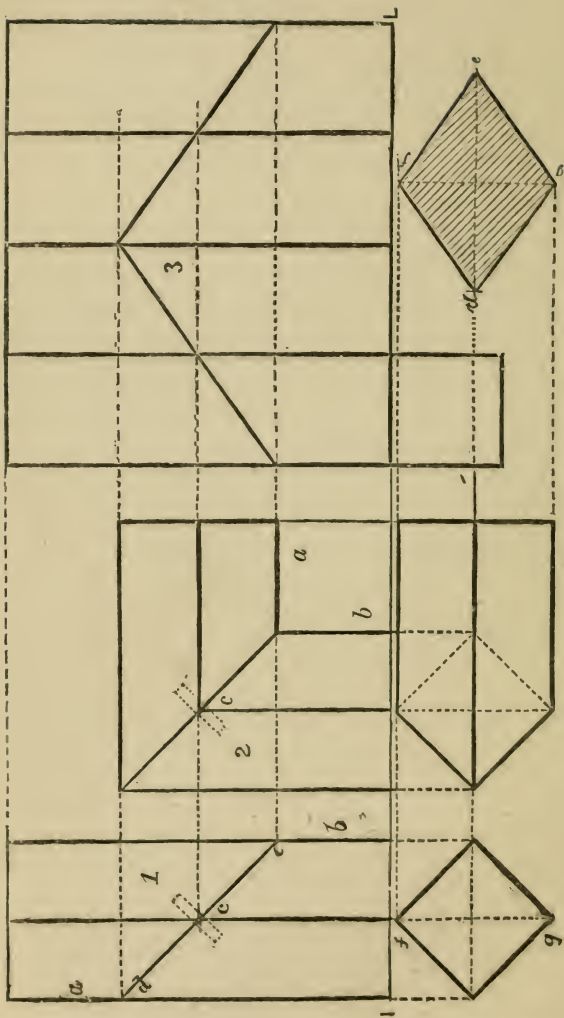
Plate X.

Plate XI.

Fig. 1. shows the plan and elevation of a piece of a square wooden pipe, when the plane of the section, instead of passing from angle to angle, as in the last plate, passes from side to side, so that the section will be a rectangle, the length of which will be equal to *c d*, and the width to *e f*, instead of a lozenge form, as in the former case.

Here, too, the upper portion may be rotated on a centre, so as to join in a right angle.

Fig. 2 is a projection of the object when placed at an angle to the vertical plane.

Fig. 3 is the development, with the shape of the section attached. It will be seen that this form will give both parts of the object, the only difference being, that in the portion formed by the fine lines the joint or seam will be in one of the edges at the *back*, whilst in the other it will be in the *front*.

Fig. 4 shows how this form is applied in constructing a common sheet-iron coal-scuttle, the lid being the covering of the section.

Plate XII.—Projection on the Inclined Plane.

On referring to Plate VII., it will be seen that the cube there represented is placed with its faces at 45° to the vertical plane, the line of the diagonal of the plan being parallel to the vertical plane. Plate XII. shows the mode of projecting views of objects, at whatever angle they may be placed in relation to both planes.

Let it be required then to project the cube when its faces are at 30° and 60° to the vertical, and when it stands on a plane inclined at 26° to the horizontal plane. It may here be pointed out, that in projecting *views* it is necessary to raise the objects at one side, or to place them on inclined planes ; for otherwise, as they are supposed to be exactly on the level of the eye, the elevation only (as in Fig. 1, Plate VII.) would be seen, but when raised at one side the top becomes visible.

Place the plan, Fig. 1, at the required angles. Draw the line A at 25° to the intersecting line. This line repre-
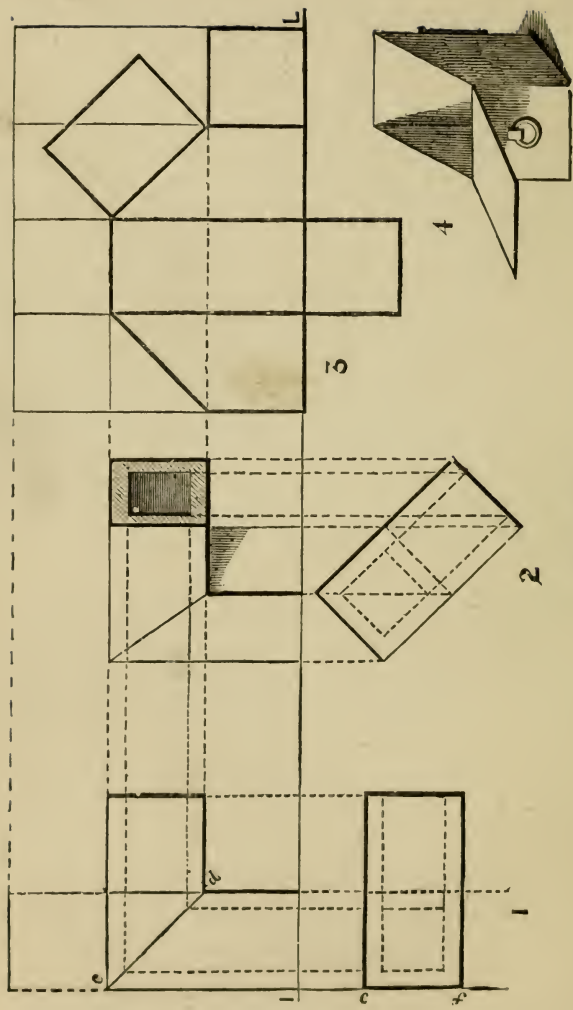
Plate XI.
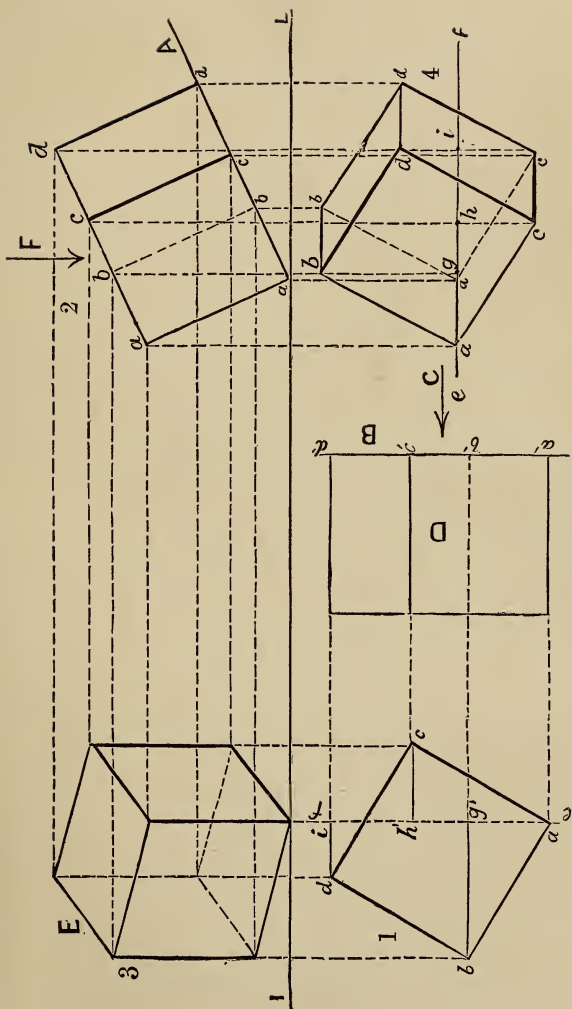
Plate XII.
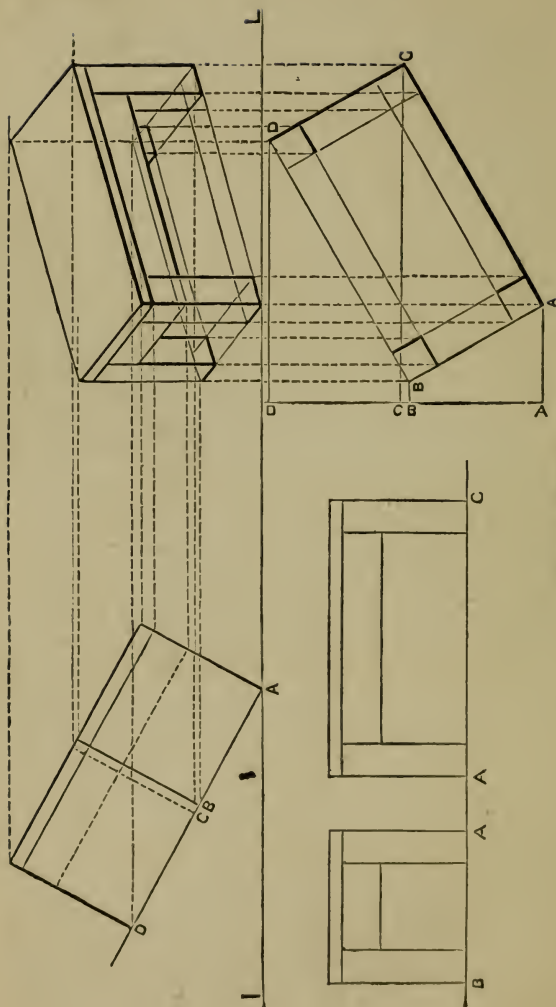
Plate XIII.

FIG. 1.

FIG. 2.

Plate XIV.

D
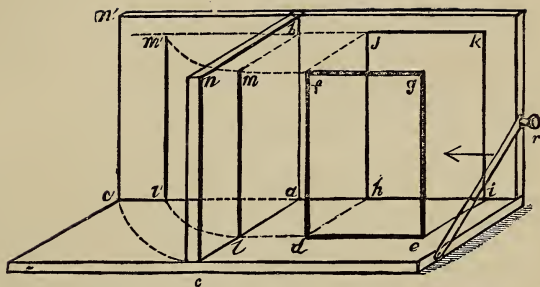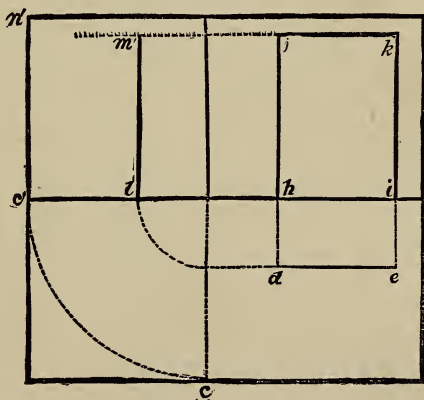
it will then be seen that the height of l' m' is the same as h j, and that in its motion the point l will have travelled through a quarter of a circle.

If now the vertical and horizontal plane be converted into one flat surface, by withdrawing the pin r, the plan, and the front and end elevation will be found to be those represented in Fig. 2.

Plate XV.

Fig. 1 is the end elevation of a triangular prism when lying on one of its long sides, its edges being at right angles and its end parallel to the vertical plane; thus the exact shape of the end—that of an equilateral triangle—is presented to view. But when the prism is turned so that the plan is at an angle to the vertical plane (Fig. 2), the elevation becomes materially altered; for as the object has rotated on the point b, a has receded, whilst f has advanced, and the apex, d, of the opposite end, which in Fig 1. was hidden beyond c, now becomes visible, the height, however, remains the same as in the original figure.

Fig. 3.—Here the plan a b c f is further rotated until its edges are parallel to the vertical plane: the elevation is then shown in the dotted parallelogram, b c d e. Let us now raise the prism at one end, so that its under side is at 20° to the horizontal plane. In this case it will be seen that the end prism rests upon the line e f, and the points of the end elevation will now become visible in the plan, and if this plan be turned (as at Fig. 4) at an angle (say 45°) to I L, perpendicular lines from the points of the plan intersected by horizontal lines from the elevation, will give the projection of the object at a compound angle.

Plate XVI.

Fig. 1 shows the plan and elevation of four such prisms meeting at a point, a figure which often occurs in designing or drawing roofs of houses, churches, &c.

The plan is formed of two figures similar to the plan of the last prism, crossing each other at right angles, and from this the elevation is easily projected.

Fig. 2 shows the projection of the object when placed at an angle to the vertical plane. And Fig. 3 is the development of one of the four parts of which the model is

Plate XV.

D 2
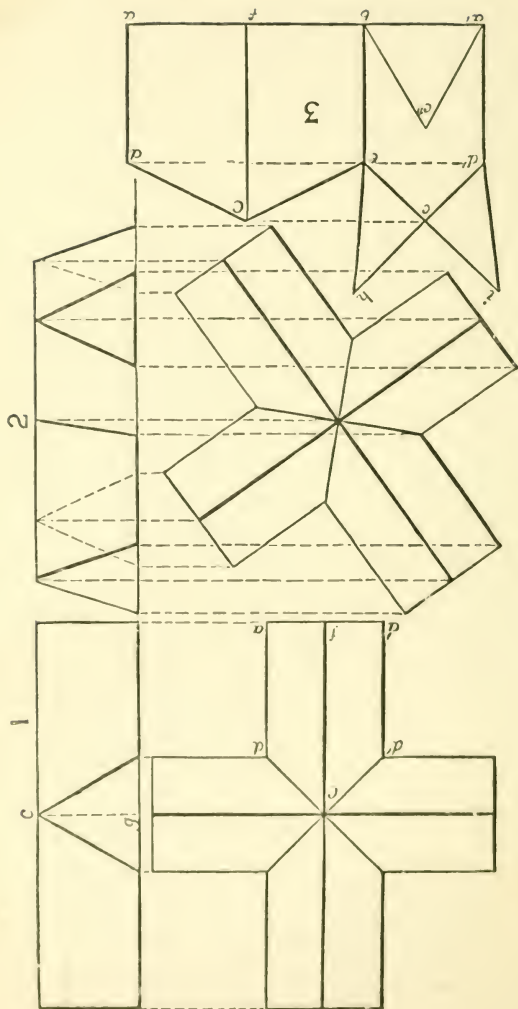
Plate XVI.

composed. To construct this development on a straight line, set off three spaces equal in width to the sides of the prism, *a' b f a*, and erect perpendiculars from the points. Make these perpendiculars equal to the lines similarly lettered in the plan. Now it will be clear that when two parallelograms, like those forming the plan of the prism, cross each other, they will form four right angles at the centre. Therefore, at *d'* and *e* construct angles of 45° which will meet in *c*, and form the required right angle, and this will complete the under side. Draw *c h* and *c i* at right angles to *e c* and *d c*, and equal to the altitude of the triangle *g c* in Fig. 1. Join *d' i* and *h e*. Then the right-angled triangles, *d' c i* and *e c h*, turned up at right angles to the mitre; *i* and *h* will then come together. The triangular end, which is represented as bent down, being now turned upward at right angles to the under side, the two upper sides are to be bent over. Then C will meet *h i*, *f* will meet *c''*, and *d a* will meet *d' a'*.

Plate XVII.—The Projection of Polygons.

In this plate the mode of projecting a plane pentagon is shown.

Fig. 1.—Let A B C D E be the geometrical figure when lying flat on the horizontal plane with one edge, A and B, at right angles to the vertical plane; as in all the planes in a similar position, the elevation would be merely a line marking the greatest width, as A C E.

Now let it be required to construct the plan of this figure, when the plane resting on A B is raised to an angle of 30° to the horizontal plane. Then as each of the points C D and E will travel through portions of circles, draw a line at 30° at A on the intersecting line, and from A, with radius A C and A E, describe arcs cutting the line in points correspondingly lettered. This, then, will be the elevation. From *e* draw a perpendicular cutting E *f* in *e'*. From C and D draw lines parallel to E *f*. Then perpendiculars drawn from $\frac{d}{c}$ in the elevation will cut these last-mentioned lines in *c' d''*. Join B *d'*, *d' e'*, *e' c'*, *c'* A, which will complete the plan of the figure.

Fig. 2.—Here the plan is turned so that *a' b'* is at 45° to the vertical plane, and it will be seen that by drawing
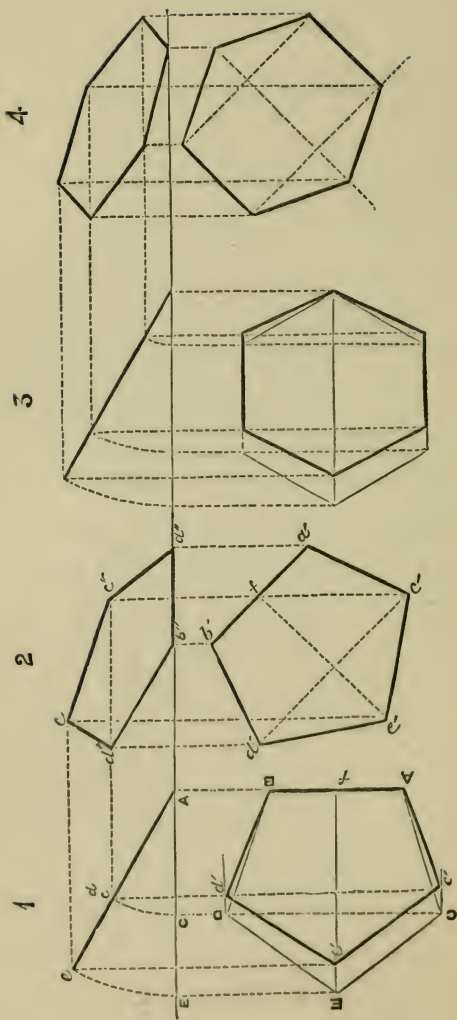
Plate XVII.

perpendiculars from the angles of the plan, and intersecting these by horizontals drawn from the corresponding points in the elevation, the projection of the plane will be obtained. It must be remembered that the pentagon being "regular,"* a line joining C and D will be parallel to A B, and will remain so, however much the plane may be raised. Thus, this line c' d' is represented in the elevation by the point $\frac{d}{c}$—the line itself being *horizontal* and at right angles to the vertical plane. (See Plate I., No. 2.) Now when the object is turned round (Fig. 2), this horizontal line becomes visible, and the perpendiculars from c' d' intersecting it, give the points c'' and d'', and it will then be seen that as the line joining these points was horizontal, and parallel to A B in the previous figure, it will remain so in the projection; and this will explain the cause of *two* points in the plan coming on *one* line in the projection—a case which will frequently occur in the projection of polygons.

Figs. 3 and 4 show the same process adapted to a regular hexagon, when resting on one of its angles, which it is expected the student will be able to work out without further instructions.

Plate XVIII.—To project a Pentagonal Prism.

Let A B C D E (Fig. 1) be the plan, and e $\frac{d}{c}$ a, e' $\frac{d'}{c'}$ a the elevation when one of the long faces is at right angles to the vertical plane. Fig. 2 is the elevation, looking directly at the point E. The mode of obtaining this elevation has been shown in Plate XIV. The upper end of the axis is shown at g in the centre of the plan,† and its position in the elevation at f g. Now it will be remembered that the ends of a right prism are equal and similar planes, parallel to each other,‡ these ends being united by lines at right

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* A "regular" pentagon is one which has all its sides and angles equal. (See volume on "Linear Drawing.")

† To find the centre of a regular polygon.—Bisect two of the angles or sides which adjoin each other, and the point where the bisecting lines meet will be the centre. (See "Linear Drawing.")

‡ When the planes forming the ends of the prism are at right angles to the long sides (that is, so that if the prism stands on one of the ends, the long sides may be vertical), it is called "a right prism." When the planes of the
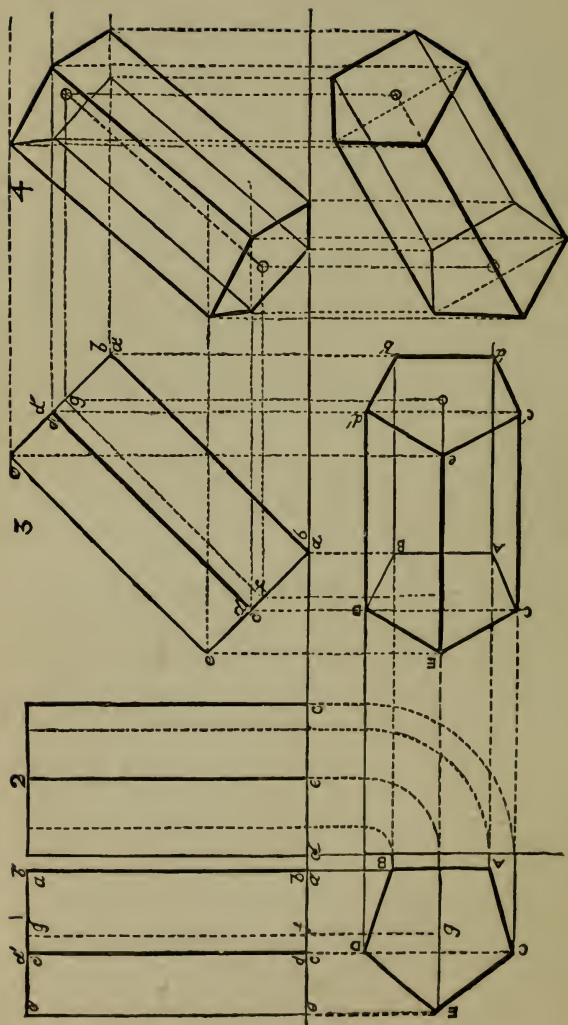
Plate XVIII.

angles to their surfaces, and it will therefore be evident, that projecting a prism is only repeating the process of projecting a plane. Thus (Fig. 3), let it be required to draw the plan of the prism when resting on A B, its axis at 45° to the horizontal, and parallel to the vertical plane. It has already been shown that the axis is parallel to the edges of a prism; consequently, as the axis is at 60°, so will be the edges. Therefore, place the line *a a* at 45°, and on this line construct the elevation of Fig. 1; project the ends by dropping perpendiculars from the points in the elevation, Fig. 3, and intersecting these by horizontals from the corresponding points in the plan of Fig. 1. Unite the points of these two plans by lines representing the long edges of the prism, which will then be seen to be parallel to the vertical plane.

Fig. 4 shows the prism when the axis is at 45° to the horizontal and 30° to the vertical plane. In this figure it will only be necessary to place the plan of Fig. 3 at the required angle with the intersecting line; then perpendiculars drawn from the angles, intersected by horizontals drawn from the corresponding points in the elevation, will give the projection.

Plate XIX.

Shows a similar application of the projection of a hexagonal prism when resting on one angle; and it is hoped that the student will be able to accomplish this without instructions.

Of Pyramids.

A pyramid is a solid which stands on a triangle, square, or polygon, and terminates in a point, all its sides being, therefore, triangles.

The axis of a pyramid is the line joining the centre of the base to the point (called the apex). When the axis rises from the centre of the base, and is perpendicular to it, the sides will be all equal triangles, and the solid is called a right pyramid. When the axis is not at right angles to the base, the pyramid is called "oblique."

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ends are slanting to the length of the prism, it is called "oblique." In this work all prisms are assumed to be "right," unless otherwise expressed.
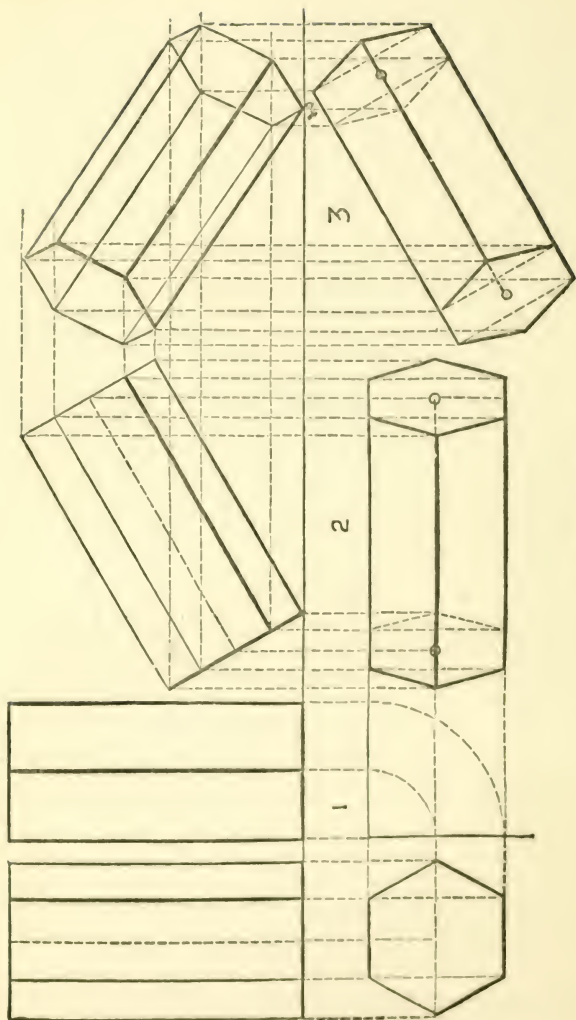
Plate XIX.

Plate XX.—The Projection of Pyramids.

Let it be required (Plate XX., Fig. 1) to project a pyramid whose base and sides are equilateral triangles. This solid is called a "Tetrahedron."

A B C is the plan, and *c a* the width of the elevation. The point requiring consideration is: How high will this pyramid be? Now, although the real shape and size of the side lettered A D B in the plan is *a′ d′ b′* (the equilateral triangle shown in the upper corner), still the line *e′ d′* would only be the height *if the three triangular sides stood upright* on their edges on the lines A B, B C, C A. *But they do not.* They slant inward until they all meet in a point; which, as they are all equal, will be directly over the centre of the plan (point D). This knowledge enables us to find the exact position of the apex. Draw a perpendicular from D, and another at *a*. Mark on perpendicular *a* the *real* height of *e d* (one side of the pyramid standing upright), viz., *a d′*. This perpendicular will be the edge elevation of the side. Then from *a*, with radius *a d′*, describe an arc, cutting the perpendicular drawn from D in *d″*, which will be the apex. Join *a d″* and *c d″*, which will complete the elevation.

It will, perhaps, at first seem strange that although we *know* the apex to be over the middle of the solid, yet it does not appear to be so in this elevation. The reason of this is, that the length of *a d″* is the projection of the height (or altitude) *e d* of the triangle, whilst the line *c d″* is the projection of the *edge c d*, which it will be seen is longer and slants more than *e d*. This appears plainly in the plan, where C D is longer than D *d*. Yet D is in the centre. The student is now advised to turn the plan, so that the edge A B may be parallel to I L: and in the elevation projected from this figure, the apex will be over the middle of the base.

Fig. 2 is the plan and elevation of a square pyramid when the two edges, A B and C D, of the base are parallel to the vertical plane. It will be seen that the position of the apex is found in the same manner as in the last figure, by marking on a perpendicular the real altitude of the side, and then inclining this elevation of the side until it cuts the axis in *e*.

Fig. 3 shows (in fine lines) the plan and elevation of
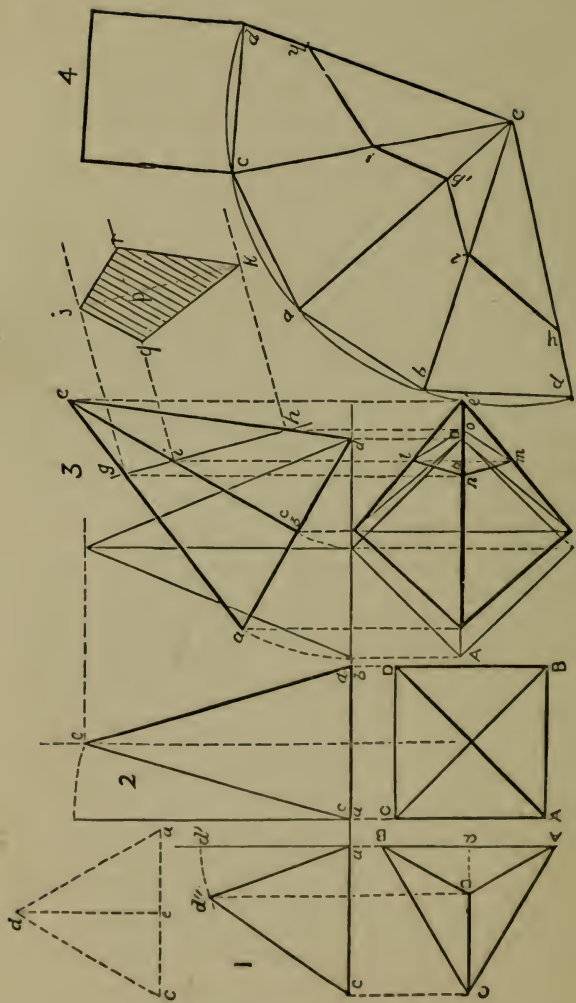
Plate XX.

the same pyramid when one of the diagonals of the plan, A D, is parallel to the vertical plane. Let it be required to draw the plan of the pyramid when resting on one angle, D ; the plane of the base being at an angle of 30° with the horizontal plane. Place the elevation at the required angle, and project the square which forms the base of the pyramid in the manner shown in Plate VI. Produce the diagonal A D, and drop a perpendicular from the apex (*e*) to cut this line *e'*. This will give the plan of the apex. Join this point to the points of the plan of the base, and the plan of the solid will be completed.

To find the true shape of the section on the line *g h*. Draw lines from *g h i* at right angles to *g h*, and draw *j k* parallel to that line ; *j k* will be the length of the section. From *g i h* draw perpendiculars cutting the plan in *l m n o*. Join these points, and a *plan* of the section will be obtained. On each side of *p'*, in the upper figure, set off the length *p m* or *p l* of the plan—viz., *p q* and *p r*. Join *j q, j r, q k*, and *r k*, and the true section will be completed.

To draw the development of this pyramid (Fig. 4). With radius equal to one of the *edges* of the pyramid, describe an arc. Draw the radius *e a*. From *a*, mark on the arc the lengths *a c, c d*, and *a b, b d*, equal to the sides of the base. Join these points, and from each of them draw lines to *e*. On either of the lines, such as *c d*, construct a square for the base of the pyramid, and this will complete the development.

It remains, however, to mark on this development the line of section—that is, the line in which the material is to be cut—so that, when folded, the frustrum (that is, the portion remaining of the pyramid if the upper part be cut off) may be formed. To find this line, mark on *a* the length *a g* of the elevation ; on *c* and *b* mark the lengths *c i* and *b i*, and on *d d* mark the length *d h*. These points joined will give the required line of section.

When the upper part of a pyramid or cone is cut off, the solid is said to be "truncated."

Plate XXI.

Fig. 1 is the plan and elevation of a hexagonal pyramid when two of the sides of the plan, B C and E F, are at right angles to the vertical plane, and its axis
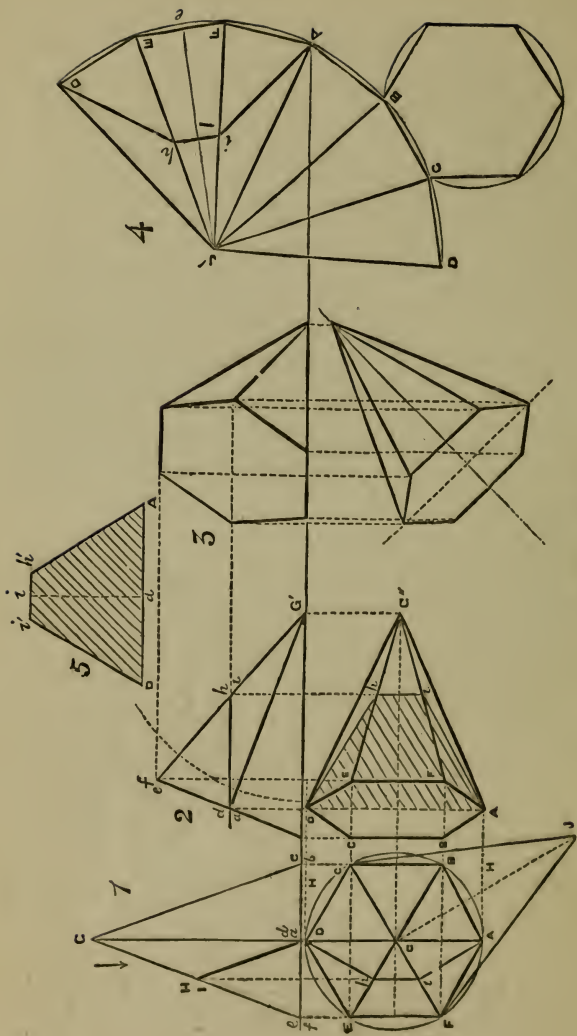
Plate XXI.

vertical. Now let it be required to draw the plan of the pyramid when lying on the side B C G. The elevation (Fig. 2) will be precisely the same as in Fig. 1, altered only in position. It will be self-evident that if a pyramid stood on a plan, and, whilst resting on the line B C, it were gradually turned over until it should lie on one of its triangular faces, the widths F E, B C, and A D would remain the same, notwithstanding the change of position; for, supposing pieces of board were placed upright on the lines H H, the angles A D would touch these "wooden walls" throughout the movement; but this is not so with regard to the widths from E to C and from F to B, which are altered according to the position of the plane of the base in relation to the horizontal plane.

The points for the plan in Fig. 2 will therefore be found by producing the lines from E C, F B in the plan, and intersecting them by perpendiculars from the corresponding points in the elevation. A line drawn from G in the plan parallel to the intersecting line, intersected by a perpendicular from G' in the elevation, will give G″, which will be the plan of the apex.

Fig. 3 is the projection of the pyramid when lying on one of its faces, with its axis at 45° to the vertical plane. In order to test the student's comprehension of the foregoing lessons, this figure is left unlettered.

It is now required to find the true shape of the section $a\,d$, H I. It will be evident that, as $a\,d$ in the elevation represents A D in the plan, A D will be the width of the section at its base. Therefore, draw A D (Fig. 5), and erect a perpendicular at its centre. Make this perpendicular equal to $\frac{d}{a}\frac{I}{H}$, and draw a line through i parallel to A D. From $\frac{I}{H}$ (Fig. 1) draw a perpendicular cutting the radii F and E of the plan in $\frac{i}{h}$. Join $h\,i$, i A, h D. Then A D $i\,h$ will be the plan of the section, or the view of it looking downward in the direction of the arrow. On each side of i, in the true section (Fig. 5), set off half the length of the line $i\,h$ in the plan—viz., $i'\,h'$. Join h' A' and i' D, which will complete the form of the section.

The next step is to develop the covering of such a solid. It is hoped that, after the instructions already given, this will prove an easy task.

From G in the plan (Fig. 1) draw a line, G J, perpendicular

to F C, and equal to the height of the pyramid ($\frac{d}{a}$ G).
Draw F J, C J, which represents the section which would
be bounded by a diagonal of the base and two of the
edges (*not sides*) of the pyramid. With J F as radius,
describe an arc (Fig. 4), and set off on it the lengths
equal to the sides of the base. Join all these points to
each other and to J'. On C B, or any other of the sides,
construct a regular hexagon, which will complete the de-
velopment of the pyramid and base. Bisect E F by the
line $\frac{J}{e}$, and on this line set off the height, e I, in the eleva-
tion (Fig. 1), and through I draw i h. Join these points
to each other and to A D ; this will give the section-line
marked in the development.

Of the Projection of Circles and Cylinders.

We now approach a branch of our subject which is of
especial importance to engineers and metal plate-workers—
namely, the projection of Circles and Cylinders, and their
development. As, however, the previous lessons have
gradually led up to this point, it is hoped that the student
will have been so prepared for the subsequent studies that
he will find but little difficulty in them.

Plate XXII.

Fig. 1 is the front elevation of a circular plane ; and it
will be seen that the plan of this is a mere line, A B, equal
to the diameter of the circle. (The aperture in a child's
money-box is the plan of the penny which drops through
it). To prepare this disc for projection, divide its circum-
ference into any number of equal parts, as a b c, and from
these points drop perpendiculars to cut the plan A B in
the points similarly lettered. If, now, we rotate the disc
so that its plan is at right angles to the intersecting line
(Fig. 2), the elevation, too, will be a line, c c', equal to the
diameter. To project this circle, transfer the points C, D D,
and E E to plan A B, Fig 2. Let it then be required to
find the forms of elevations when the plane of the disc is
at 60° and 30° to the vertical plane. Place the plan at
each of these angles, B' B″. Taking A as a centre, de-
scribe arcs from the points in the plan to cut the plans B'
and B″ in C' D' E'. From each of these points draw

Plate XXII.
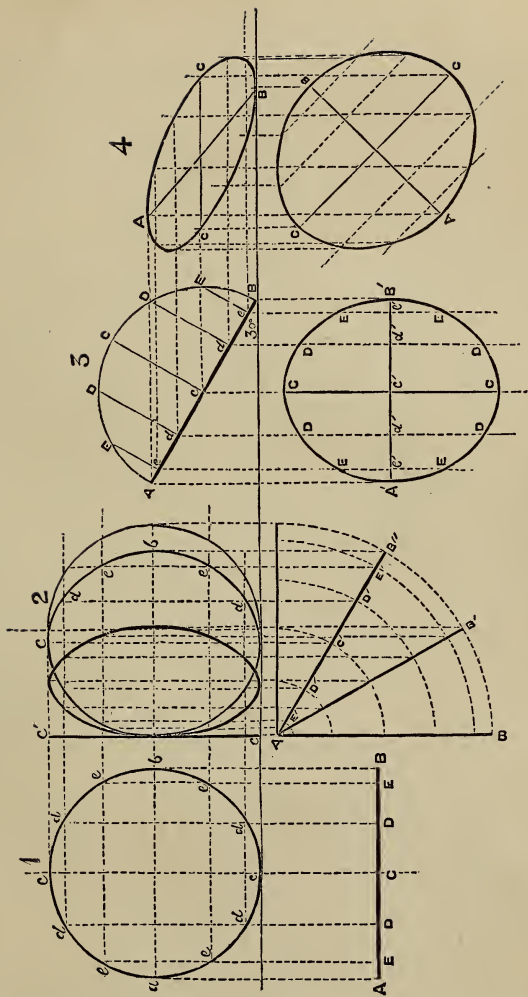
perpendiculars ; and from the points similarly lettered in
the elevation, draw horizontals. The intersections of
these two sets of lines will give the points *c d e*, &c.,
through which the curve is to be drawn by hand in the
first instance, but it may subsequently be inked by means
of the French curve, or centres may be found from which
parts of the ellipse may be struck.

The principle on which the projection of a circle is
founded having thus been shown, Fig. 3 gives a simpli-
fied method. Let it be required to draw the plan of a
circle when resting on one end of a diameter which is
parallel to the vertical plane, the surface being at 30° to
the horizontal plane. The line A B, placed at 30° to the
intersecting line, will then represent the elevation of the
disc. From the centre of this line, with the radius of the
circle it is intended to project, describe a semicircle, and
divide it into a number of equal parts, C, D D, E E. From
each of the points draw lines meeting A B at right angles
in the points, *c d d e e*. Draw any line parallel to the
intersecting line, and draw perpendiculars to it from A
and B ; then this line A' B' will be the plan of the
diameter which is parallel to the vertical plane. The
semicircle drawn on A B represents one-half of the disc
lifted up until it is parallel to the vertical plane. The
lines C D and E thus show the distance which each of
these points in the circumference is from the diameter
A B. Therefore, from *e e*, *c*, *d d* in the elevation draw
perpendiculars passing through the plan of the diameter
A' B' in *e' d' c' d' e*. From these points set off on the lines
drawn through them, and on each side of A' B the lengths
e E, *d* D, *c* C, &c., and through the points thus obtained
the plan is to be drawn.

Fig. 4 shows the mode of projecting a circle when its
surface is at 30° to the horizontal and one of its diameters
at 45° to the vertical plane. Place A B at 45° to the
intersecting line, and on it construct the plan by measure-
ment from Fig. 3. This is best done by drawing a line,
C C, at right angles to the diameter, A B, and on each
side of the intersection marking off the distances, *e e*, *d d*.
By drawing lines through these points at right angles to
A B, and making them the same length as in the plan
No. 3, the points for the present figure will be obtained.
From these points in the plan draw perpendiculars, and

from the points correspondingly lettered in the elevation, draw horizontals, and the intersections will give the points through which the projection of the circle is to be drawn.

Of Cylinders.

A cylinder is a solid body of the character of a prism, but having its *ends circles.* The axis, or line on which a cylinder might be turned, unites the centres of the ends ; and if the ends are at right angles to the axis, the solid is called a *right* cylinder. If the ends are at an angle with the axis, so that if the cylinder were placed on one of them it would be slanting instead of upright, it is called an *oblique* cylinder. In the first case, the ends would be circles ; but in the second, although all the sections at right angles to the axis are circles, the ends being at an angle to it are ellipses. It will be readily understood that all sections passing from one end of a cylinder to the other, *parallel to the axis,* will be parallelograms ; and by rolling up a rectangular piece of paper, it will be seen that the surface or development of a cylinder is a parallelogram, the height of which is equal to the length of the cylinder, and the breadth to its circumference.

Plate XXIII.

Fig. 1 is the plan and elevation of a cylinder when standing on its base, and it will be evident that then, although the cylinder might be rotated on its axis, that axis would remain at right angles to the horizontal and parallel to the vertical plane.

Fig. 2 shows the elevation of the cylinder when its axis is at 45° to the horizontal, and parallel to the vertical plane. To project the plan of this, on A B describe a semicircle which will represent half of the end. Divide this semicircle into any number of equal parts, E, D, C, &c., as in Fig. 3 in the last plate. From these divisions, draw lines parallel to the axis of the cylinder, which, passing from end to end, will give the same points in both. Draw a line for the axis of the plan parallel to the intersecting line, and perpendiculars from the various points in the elevation. Mark off the lengths C C, &c., on each side of the axis, as in Fig. 3, Plate XXII., and

E 2
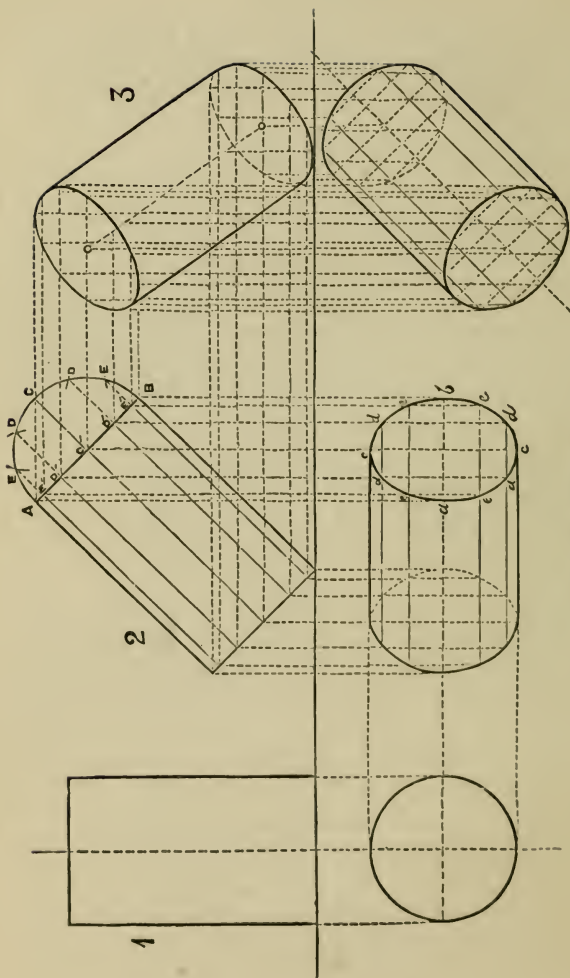
Plate **XXIII.**

through the points thus obtained draw the ellipses forming the plans of the ends. Unite these by lines parallel to the axis, which will complete the plan.

Fig. 3 is the projection of the cylinder when the axis is at 45° to both of the planes of projection. No description of the working is deemed necessary, as it is simply a repetition of Fig. 4 in the last plate, and will no doubt be readily understood.

Plate XXIV.

On referring to Plate X., the student will be reminded that, if a solid be cut across, the parts will, when rotated on a centre, form an "elbow"—that is, they may be joined so as to turn a corner. This principle holds equally good in relation to cylinders.

Fig. 1 is the plan and elevation of a cylinder which it is required to cut so that the parts may be joined to form an angle of 90°. The following rule must be impressed on the minds of students — viz., Whatever may be the required angle, the section must be made at *half* that angle with the axis. Thus, if a pipe is to follow two walls which meet at an angle of 120°, each part must be cut at 60°. And, therefore, in the present figure, draw the section-line A B at 45° (half of 90° required). If now the upper part of the cylinder be rotated on a centre (C), the point B will meet A, and the line B F will become A G. Now divide the plan into any number of equal parts, E D, &c., and carry up perpendiculars from these points to cut the section-line in d' d' e' e'.

To find the true section, draw A B, Fig. 2, equal to the section-line A B in Fig. 1, and set off on this line all the distances, e', d', &c. Through these points draw lines at right angles to A B, and set off on them, on each side of the line A B, the distances which the points similarly lettered are from the central line, A B in the plan, thus obtaining the points C c, D d, E e. Draw the curve of the ellipse, which forms the true section, through these points.

Fig. 3.—To develop this cylinder, draw a horizontal line and a perpendicular, A. On each side of A set off the six equal spaces into which the two parts of the plan in Fig. 1 are divided—viz., E D C D E B, and e d c d e B, placing
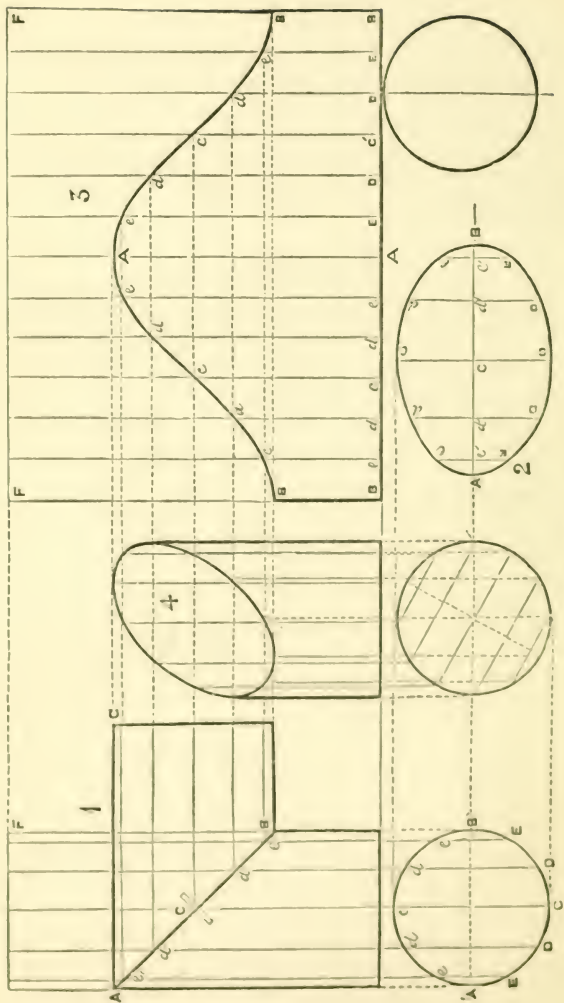
Plate XXIV.

the letters in the order in which they follow on each side of A. Erect perpendiculars from each of these points, making B F equal to the height of the original cylinder, Fig. 1. Join F F, and the parallelogram B F F B will be the development of the entire cylinder.

To trace on this development the section-line—that is, the line in which the material is to be cut so as to form both the parts of the cylinder, erect perpendiculars from each of the points between B B, and make them the height of those similarly lettered in the elevation. This is best done by drawing horizontals from the points in the section-line to cut the perpendiculars in the development which are similarly lettered. The points A, *e e*, *d d*, *c c*, *d d*, *e e*, B B, will be thus obtained ; and through these the curve is to be traced, which will be the development of the line of section ; and if a piece of sheet iron, or any other material, were so cut, the parts when rolled and joined will give exactly the same figures, the joint or seam being at the highest point in the one, and at the lowest in the other part.

Fig. 4 is a view of the lower portion projected from the plan, when the diameter, A B, is at an angle instead of being parallel to the vertical plane.

Plate XXV.

Fig. 1 shows the elevation and half-plan of a cylinder, which, on being cut twice at 45°, may be converted into a " double-elbow."

Having drawn the half-plan (only the half is required, as the points in the other half would be immediately at the back of those here given), project the elevation from it ; then divide the semicircle into any number of equal parts, and from these points of division draw perpendiculars. Next draw the section-lines at the required angle (in this case 45°), and at the two extremities of the lower one draw lines at right angles to the elevation of the cylinder ; make these lines equal in length to the middle portion of the cylinder, and join them by a line at 45°. Erect perpendiculars at their extremities equal respectively to the corresponding lines in the lower portion of the object. Join these by a horizontal line, and this will complete the elevation.
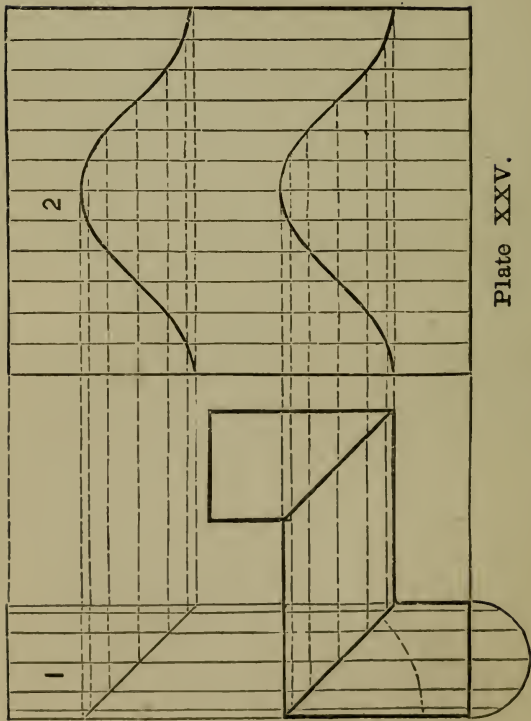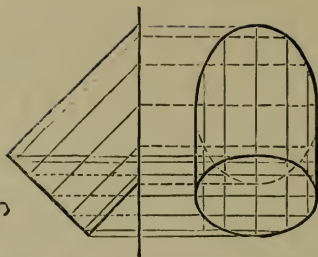
Plate XXV.

The development of the piece of metal of which this double-elbow is to be cut must now receive our attention. Produce the base-line of the cylinder, and at any part of it erect a perpendicular, from which set off on each side the same number of equal parts as that into which the half-plan of the cylinder is divided, and draw perpendiculars from the points. From the top of the original *erect* cylinder draw a horizontal line, and this, uniting the two external perpendiculars, will complete the general development. Returning now to the elevation of the original cylinder, it will be seen that the perpendiculars which were drawn from the points of division in the plan, cut both of the section-lines, and from these points of intersection draw horizontal lines to cut the perpendiculars in the development, that drawn from the highest point (viz., the point where the section-line starts from the side of the elevation) to cut the *central perpendicular*, and that from each of the other points to cut the next *pair* of perpendiculars in succession; through these points the curve, which is the development of the section-line, is to be drawn. The lower section-line will of course be developed in precisely the same manner from the corresponding points of intersection occurring on it.

Fig. 3 is the elevation and plan of one of the ends of the above object when resting on its section. On a horizontal line mark off the length of the *section-line in the elevation*, and at the extremities draw lines at 45°; make these equal to the length of the longer and shorter sides of one of the end pieces of the elevation, and join them by a line which will (if their lengths be correct) be at right angles to them. Divide this line into two equal parts, and from the bisecting point draw a line parallel to the sides already drawn : this will be the axis. On each side of this draw lines parallel to it, and at distances apart corresponding with those in the elevation, to meet the intersecting line. Drop perpendiculars from these intersections, passing through a horizontal line drawn in the lower plane ; on these set off from the horizontal line the widths of the corresponding lines in the plan ; join the extremities by tracing an ellipse to touch each, and this will be the true section on which the object now rests. From each of the points through which the ellipse has been traced draw horizontal lines, and intersect these by

perpendiculars drawn from the points occurring in the *end* of the object; through these intersections draw the ellipse, which will be the plan of the end.

Plate XXVI.—Plan and Projection of a Speed-pulley.

This is a further application of the lesson illustrated by Plate XXIII., and consists of the projection of three pairs of parallel circles. Having drawn the plan, describe on A B a semicircle, and divide it into any number of equal parts—*c d e f g*. From each of these points, draw lines meeting A B at right angles in *c' d' e' f' g'*. These lines will cut C D in *c" d" e" f" g"*. Draw a line, X X, at a height above I L equal to the radius of the circle—viz., *e' e*. From A B, and all the points between them, draw perpendiculars passing through X X, and on these perpendiculars, set off on each side of X X, distances corresponding to the distance between the point similarly lettered in the semicircle, and the line A B, as *e e', d d'*, &c., and this will give the points A' B' C' D' E' F' G', through which the ellipse is to be drawn. From each of the points last mentioned, draw horizontal lines, and intersect them by perpendiculars from the points C, *d", e"*, &c., in the plan, and the intersections of the lines correspondingly lettered will give the points E", D", A", &c. The other two wheels are to be projected in precisely the same manner from semicircles equal to half their surface. The lettering of these is omitted to avoid confusion in the diagram; but the student, who is expected to work on a much larger scale, is advised carefully to letter every point.

Of Cones and their Projection.

A cone is a solid, the base of which is a circle, and the body of which tapers to a point called the apex.

The straight line drawn from the centre of the base to the apex of the cone is called the *axis*.

When the axis of the cone is perpendicular to the base, the cone is called a "*right*" cone; but when otherwise, it is called an "oblique" cone.
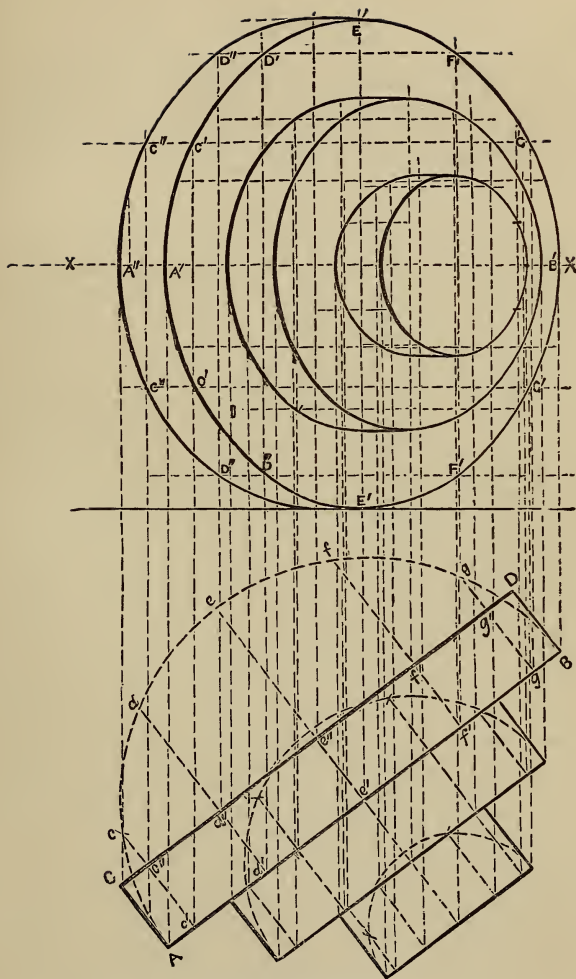
Plate XXVI.

The curved surface of a cone is equal to the sector * of a circle, the radius of which is equal to a straight line drawn from any point in the circumference of the base to the apex, and the arc-line of the sector is equal to the circumference of the base of the cone.

Plate XXVII.

Fig. 1 is the plan and elevation of a cone when standing on its base, its axis being perpendicular to the horizontal and parallel to the vertical plane. The apex is thus over the centre of the plan, and the solid is, therefore, called a *right* cone.

Fig. 2.—To draw the plan of this cone when lying on the horizontal with its axis parallel to the vertical plane, draw the elevation lying on I L. To do this, at any point in I L construct an angle similar to A B C in the elevation ; make the sides of the angle equal to those of the elevation, and join A′ C′. Bisect the angle and produce the bisecting line to D. This will be the axis. Now begin the plan by drawing C B parallel to I ‚L. From D in the elevation, with radius D A, describe a semicircle, and divide it into any number of equal parts, *e f g h.* From each of these points draw lines meeting A C at right angles, and from these points of meeting drop perpendiculars passing through C B on the points *e f* D *g h.* Set off from these points on their respective perpendiculars, and on each side of C B, the lengths of the lines between A C and the semicircle, and by this means the points through which the ellipse is to be drawn will be obtained. Join the point B to each end of the ellipse, which will thus complete the plan. The students should now, as an exercise, turn the plan so that its axis is at a given angle to I L, and then make a projection from it and the present elevation.

Fig. 3 shows the elevation and plan of the cone when resting on the extremity of one diameter of the base, the plane of which is at 30° to the horizontal plane.

It will be evident that whether the cone lies on the paper, or stands on one extremity of a diameter, so long

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* *Sector.* A part of a circle, contained between two radii and a portion of the circumference.
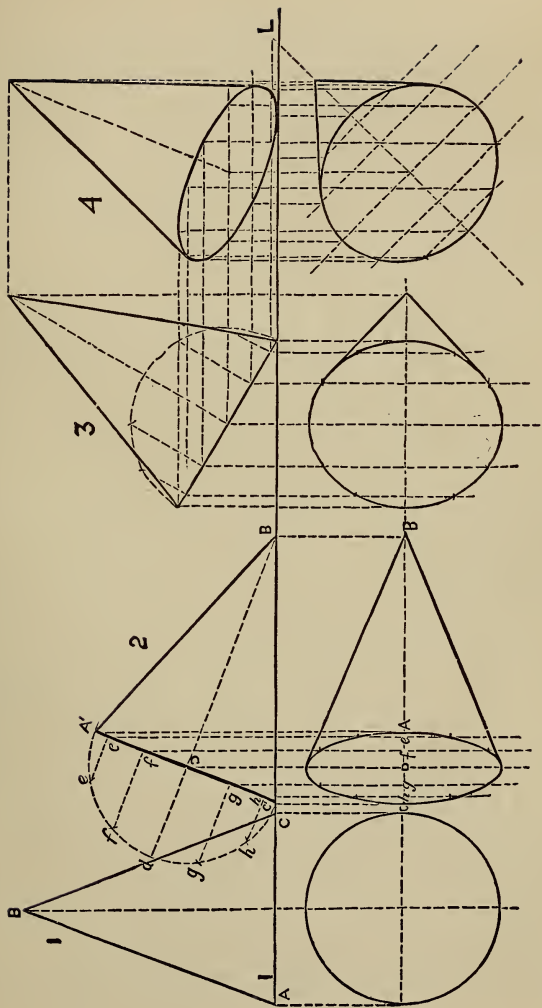
Plate XXVII.

as the axis remains parallel to the vertical plane, the elevation will be the same in *form*—changed only in position—and therefore the line which is the elevation of the base has been placed at the required angle. Construct on it an isosceles triangle of the given altitude, which will form the elevation of the cone. It must here be remarked that this figure and the next have been left unlettered, so that the student may become gradually accustomed to follow the points through their various change of position ; and, with Fig. 2 to guide him, it is thought that he will be able to complete this projection of the plan from the instructions here given.

It will be remembered, that although the base of the cone is rendered by a straight line in the elevation, that line is the *edge elevation* of a *circle* ; therefore, from the middle point in the line describe a semicircle, which will represent one-half of the base, turned up so as to be parallel instead of at right angles to the vertical plane. Now divide this semicircle into any number of equal parts, and from each of these draw lines at right angles to the diameter. Next draw a line in the horizontal (or lower plane) parallel to I L, and a portion of this line will become the axis of the cone. From each of the points in the base of the cone draw perpendiculars passing through this horizontal, and make them the same length on each side as the lines drawn from the points in the semicircle to the diameter. Through these points trace by hand the ellipse, which represents the plan of the base, being the view from a point immediately over it. Drop a perpendicular from the apex of the elevation to cut the horizontal, and this intersection will be the plan of the apex. Join this by straight lines to the widest parts of the ellipse, and this will complete the projection.

Fig. 4 is the projection of the cone when the base is at 30° to the horizontal, and its axis at 45° to the vertical plane. It has been shown in several previous figures that an object may be rotated without the height of any part of it being altered ; and thus, as in the projection now required, the base of the cone is to be at an angle to the horizontal plane similar to that of the last figure, the plan will be the same in shape, but altered in position ; therefore, repeat plan of Fig. 3, placing it so that the axis is at 45° to I L ; then draw

perpendiculars from all the points in the ellipse, and cut them by horizontals from the points in the elevation. Draw the projection of the base through the intersections. Draw a perpendicular from the point, which is the plan of the apex, and a horizontal from the apex in the elevation. The intersection of these will give the apex of the projection. Join this point to the ellipse representing the base, which will complete the figure.

Of the Sections of Cones.

If a cone be cut across, so that the plane of section may pass through the axis at an angle, and cut the slanting surface of the cone on the opposite sides, the section is called an ellipse.*

To draw an Ellipse which shall be the true Section of a Cone on a given line.

Let Fig. 1, Plate XXVIII., be the plan and elevation of the cone, and A B the line of section. Divide the circumference of the plan into any number of equal parts, as C D E F G H I, and D' E' F' G' H' I, and draw radii. Project these points on to the base of the cone, and from C" D", &c., draw lines to the apex J'. The diagram up to this point represents a cone, up the slanting surface of which straight lines have been drawn, which on looking down on the apex would appear as radii of the circle forming the plan.

The line E" J" in the elevation is therefore the radius E J in the plan, and thus, the plan of any point marked on E" J" must fall somewhere on the radius E J. Now the section-line A B cuts through all the lines drawn to the apex of the cone in the points *d e f g h*, and it will be remembered that, although in the elevation the section is represented by a single line, A B, it will assume a different form in the plan. From points A and B draw perpendiculars cutting the diameter C I in *a* and *b*, and from

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* An ellipse differs from an oval by being the same shape at both ends; but in an oval, the one end is more pointed than the other. (See " Linear Drawing," in which are also given various methods for describing conic sections as plane figures.)
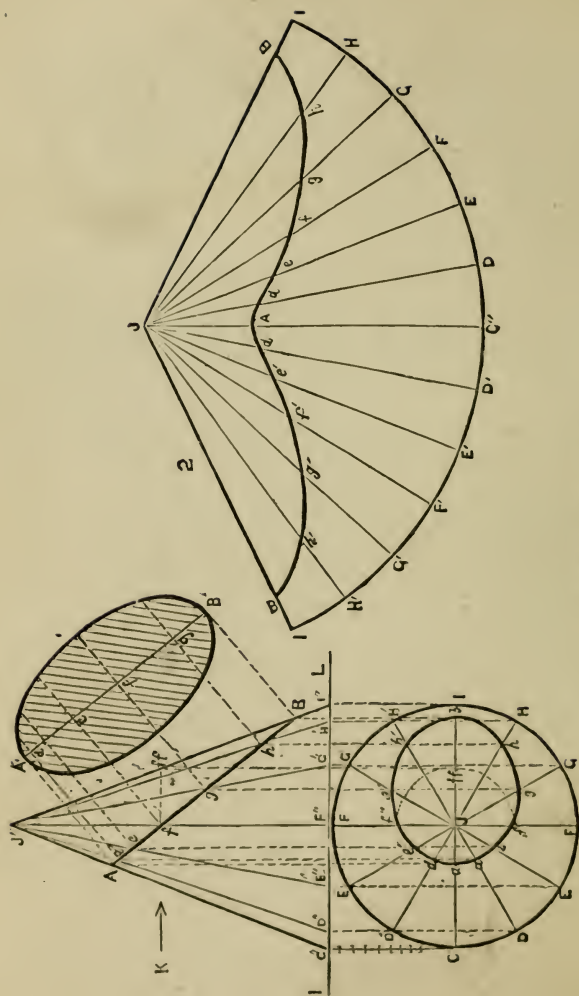
Plate XXVIII.

d e g h in the elevation, draw perpendiculars cutting the radii of the plan, which bear the same letters. Draw the curve, which will unite *e' d' a d e*, and also the curve uniting *g' h' b h g*. It will at once be seen that these two curves form the ends of an ellipse which is to be the *plan* of the section, but that a point is wanted on F and F' in order to complete the figure. But we cannot draw a *perpendicular* from the point *f* in the elevation, to cut the radius F in the plan, as we have done in the other lines, *because the radii* F F' *are but portions of the same perpendicular on which the point f is situated*, and therefore no intersection can be obtained.

Now let us remember that the line F'', though appearing perpendicular to I L when looked at in its present position, would, if looked at from K, in the direction of the arrow, be seen to be as much a portion of the slanting surface of the cone as I J, and therefore the line F J would be seen to make the same angle with the horizontal plane as I J. If therefore we rotate the cone on its axis, the point *f* will move to *f f*, and a perpendicular drawn from *f f* will give us *f f* in the plan. If now we turn the cone to its original position (which will be represented by drawing a quadrant from the centre of the plan with radius J *f f*), the quadrant will cut the radius F in *f'* and F' in *f''*. Join *e* and *g* and *e'* and *g'* by curves passing through *f* and *f'*, which will complete the plan of the section. This is not the *true section*, but the view when looking straight down upon it, and as it is slanting, its length from *a* to *b* will seem shorter than it really is. It will be evident that the true length of the section is the line A B. From these points, and also from *d e f g h*, draw lines at right angles to the section-line, and A' B' parallel to it. On each side of the points *d e f g h* in the line A' B, set off the distances which the points similarly lettered are from C I in the plan, and these will give the points through which the true section may be drawn.

To Develop the Surface of the Cone.

Fig. 2.—From any point, as J', draw a line equal to J C in the elevation—viz., J' C''—and with J' C'' as radius, describe an arc. On each side of C'' set off on this arc the equal distances C, D, E, &c., of the plan (Fig. 1). Join I I to J,

and the sector thus formed will be the development of the cone, on which it is now required to trace the line of section. To do this, draw lines to J″ from the points G′, F′, &c., marked in the circumference. From C″ set off the length C″ A of the elevation; from D and D″ set off the length D d; on the other lines set off the distances which are correspondingly lettered in the elevation, and through the points thus obtained draw the curve, which is the line in which the material would be cut so that, when rolled I B and I B are brought together, a truncated cone may be formed, the section of which on the line A B will be the ellipse A′ B′.

Plate XXIX.—The Parabola.

If a cone be cut by a plane parallel to one of the sides of the triangle which forms its elevation, the section is called a parabola.

Fig. 1.—To draw the parabola which shall be the true shape of the section of the cone A B C, on the line D E, which is parallel to C B.

Divide E D into any number of parts, as F G H, and through these points draw lines parallel to the base, meeting the sides of the triangle in f g h i on each side. Now it will be evident that all sections of a *right* cone which are parallel to the base must be circles; and therefore, as the base A B of the elevation is represented in the plan by the circle A′ B′, the line f f in the elevation will be represented by the circle $f′$ in the plan; and similarly, the lines g and h in the elevation, become the circles $g′$ and $h′$ in the plan. (Refer to Plate XXII.)

From E in the elevation draw a perpendicular which, passing through the plan, will give the line E′. E′. This is the line where the section-plane, entering the cone at D, will cut the base. A perpendicular dropped from D will mark on the diameter the plan of the top of the section—viz., D′.

An additional point, I, has been inserted between H and D, in order to gain more points for tracing the curve. This point is to be worked similarly to the others.

It has been shown that the section-plane cuts the elevations of circles f g h i in F G H I, and therefore perpendiculars dropped from these points to cut the

Plate XXIX.
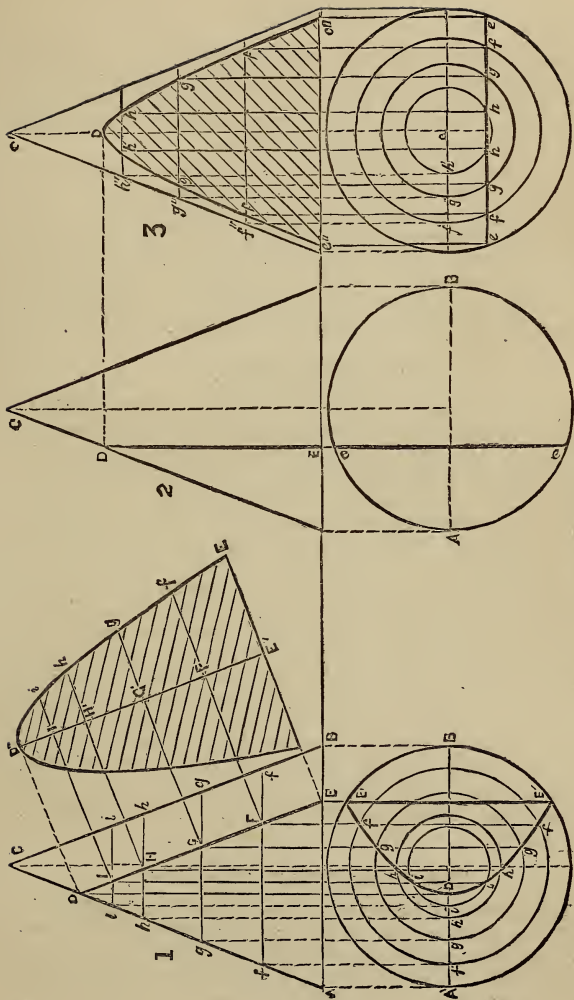
plans of these circles, will give the points *f g h i* in the
plan. The curve drawn through these points, together
with the straight line E E, forms the *plan* of the para-
bola, being the view of the slanting surface E D as seen
from a point immediately over the cone.

To draw the true shape of the section, draw a line D″ E″
parallel to D E, and from D E F G H I draw lines at right
angles to D E, passing through D″ E″ in F′ G′ H′ I′. On
each side of these points, mark on the lines drawn through
them the distances which the points E E *f g h* in the plan
are from the diameter A B—viz., *f g h*. Through these
points draw the curve, which will be the true parabola
formed by the plane cutting the cone in the line D E.

The Hyperbola.

Fig. 2.—When a cone is cut by a plane which is
parallel to the axis, the section is called the *Hyperbola*.

In this case the object of the lesson is to find the
true section of the cone, caused by a plane, of which
D E is the edge elevation, cutting it parallel to the axis.
Rotate the cone on its axis so that the section shall face
the spectator, in which position (Fig. 3) it will evidently
be parallel to the vertical plane. Now from C in the
plan, draw any number of circles, cutting the line *e e*
in *f g h*. The diameters of these circles will be marked
by the points *f′ g′ h′*. From these draw perpendiculars
cutting the side of the cone; and the lines *f″ g″ h″* drawn
parallel to the base will give their elevations. Now from
the points in the plan where the section-line cuts the
circles—viz., points *f g h*, draw perpendiculars cutting
the lines *f″ g″ h″* in *f g* and *h*; then from D, Fig. 2,
draw a line to cut the axis in D′, and perpendiculars
from *e′ e′* to cut the base of the elevation in *e e*. The
curve drawn through all these points will be the required
Hyperbola.

The Penetration of Solids.

When one solid meets another it is said to *penetrate* it,
and the development of the form generated at the inter-
section of the bodies, is a study of the utmost importance
to artisans. The lessons on this subject in the present
volume commence with those of the most elementary

character, and advance by very gradual stages. Only fundamental principles are, however, developed, in order to prepare the student for the advanced studies which will be given in the subsequent volumes adapted to the respective branches of industry.

Plate XXX.

Fig. 1 represents the plan and elevation of a square prism penetrated by another of smaller size, their axes * being at right angles to each other, and two of their faces being parallel. The figure at this stage is so simple that it requires but little explanation. The points not visible in the present view, owing to their lying exactly beyond others, are marked with letters corresponding to those on the points which are in front of them, with the addition of a dash ('), and the points themselves will become visible in Fig. 2, where the object is rotated.

Fig. 2.—Place the plan at any angle (as required). The projection will then be accomplished, as in previous figures, by drawing perpendiculars from the points in the plan, and intersecting them by horizontals from the corresponding points in the elevation. Points G and H will mark the line of penetration—that is, the line at which the smaller prism enters the larger.

Fig. 3 is the development. The widths of the sides being equal to A B, and the length to the height of larger prism, the squares represent the cavities through which the smaller prism would pass when the development is folded into a square form.

Plate XXXI.

Fig. 1 represents the plan and elevation of a square prism penetrated by a smaller one, when the axis of the latter is at an angle to that of the former. The student who has followed the lessons to this point, will find no difficulty in projecting the plan from the elevation, and by turning the plan, to project the view given in Fig. 2. The object, however, of the lesson is to show that, although the penetrating prism is *square*, the opening through which it is to pass, and which it is to *fill* up, is an *oblong*.

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* *Axes*, plural of axis.
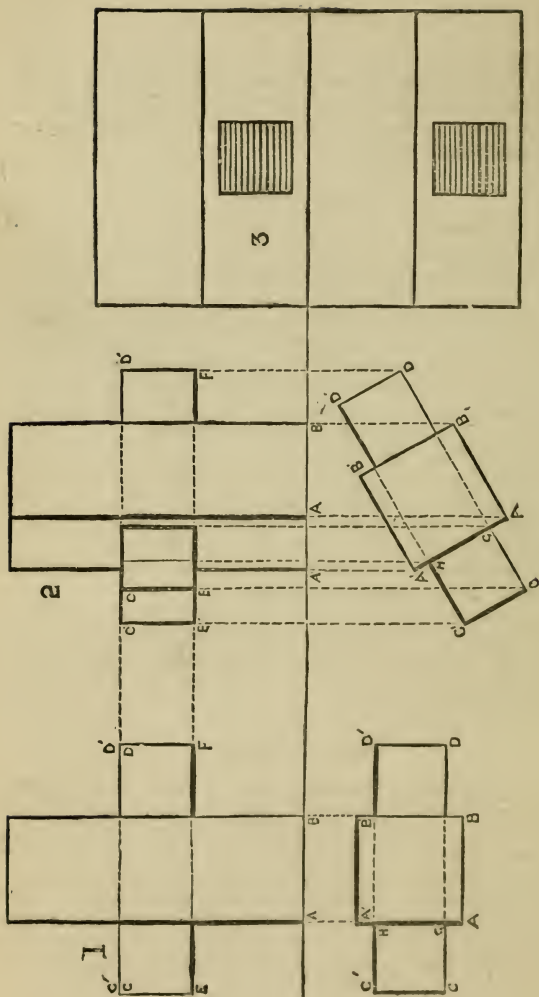
Plate XXX.
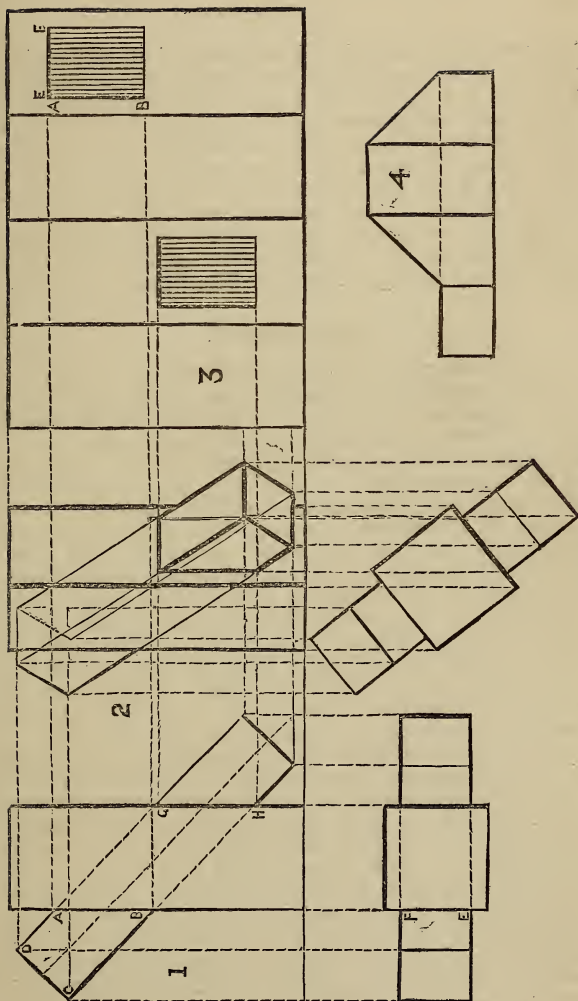
Plate XXXI.

The reason of this is, that although the width of the prism from E to F is not altered by its being placed obliquely, the line A B across the side C D is longer than E F. Therefore, having developed the surface of the larger prism, draw horizontals from A and B, which will give the top and bottom of the oblong, which is to be made of the width E F. The aperture on the opposite side is to be projected in the same manner from G H.

Fig. 4 shows the development of one of the ends of the smaller prism. Full directions for working this figure have already been given in Plate XI., Fig. 4.

Plate XXXII.

Fig. 1. is the plan and elevation of a square prism, penetrated at its *edges* by a smaller prism, their axes being at right angles to each other. Having drawn the square A B C D—the plan of the larger prism—draw the line E F through the centre, and make it equal to the required length of the smaller prism. At F draw J K, and at E draw G H, equal to the diagonal. On G H construct half the square of the end—viz., produce F E until E I equals E H, and join I H and I G. Draw H J and G K. These will complete the plan of the smaller prism, which will penetrate the sides of the plan of the larger prism in L M N O. Project the elevation C A D of the larger prism from the plan, and draw G′ K′ at right angles to the axis. On each side of G K set off the length E I— viz., points E E F F. Draw perpendiculars from L and M, cutting G′ K′ in L′ M′. Join C C L′ D D M′, which will be the lines marking the intersections of the two prisms.

Fig. 2 shows the projection of this object when the axis of the smaller prism is at an angle to the vertical plane.

Fig. 3 is the development of the longer prism, showing the shape of the openings through which the smaller prism is to pass. On a straight line set off four times the width of the side of the plan represented by A D B C A. Erect perpendiculars from these points equal to the height of the prism, and draw a horizontal line at their extremities. Produce E′ F′, G′ K′, and E F, to cut line C in P Q R, and line D in P′ Q′ R′. On each side of Q set off Q S and Q T, equal to C L in the plan, and set off the same measurement—viz., Q′ S′ and Q′ T′—on each side of Q′.
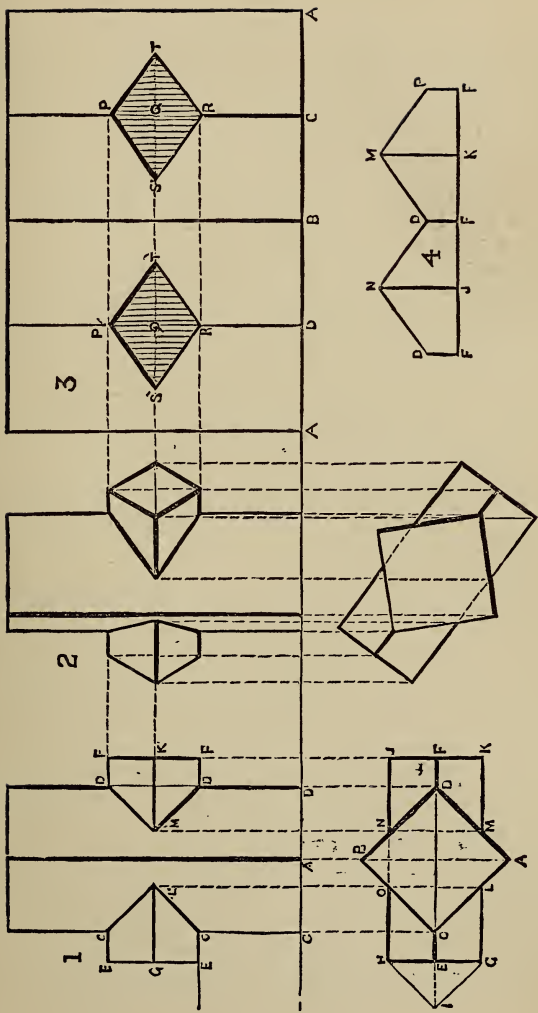
Plate XXXII.

Join P S R T, and also P′ S′ R′ T, and two lozenge-shaped figures will be formed. It will be observed that these are wider across than the prism which is to pass through the aperture, but it must be remembered that the two sides of the larger prism are bent at right angles to each other, and thus, when the perpendiculars A and B′ are brought together, S and T approach each other until the distance between them is equal to M N in the plan, which, it will be seen, corresponds with the diagonal of the end of the smaller prism.

Fig. 4 is the development of one of the projecting ends of the smaller prism. Here the widths are taken from G I in the plan of the smaller prisms (Fig. 1), and the heights from D F, J O, M K.

Plate XXXIII.—Plan and Elevation of a Cylinder penetrated by a smaller one.

The circle represents the plan of the larger, and the parallelogram D D′ E E′ that of the smaller cylinder. From this figure project the mere cross which forms the elevation. No explanation of this process is deemed necessary, the object of the lesson being to find the curve generated at the points where the penetration takes place. The student is here reminded, that as the plan is the view of the object when *looking down* upon it, the line C A B C, which is the top line of the smaller cylinder in the elevation, is the *middle* line in the plan ; and thus the line D E, which is the front, or most prominent line of the cylinder in the plan, is represented by D E, the middle line in the elevation.

From C′ in the plan, with radius C′ E, describe a semi-circle, which represents half of the plane of the end of the cylinder. This plane, although laid down flat, is supposed to stand upright on the line E E′ at right angles to the plan. Divide the semicircle into any number of equal parts, and from these divisions, draw lines meeting E E at right angles in F and G. Set off the lengths of these perpendiculars on each side of the line D E in the elevation—viz., F F and G G, and draw lines from these points across the whole length of the elevation of the smaller cylinder. Draw similar lines parallel to C C′ from the cor-

responding points in the plan—viz., F F′ G G′, which lines will be seen to pass, not only through the smaller, but also through the larger, cylinder, representing as they do planes common to both the solids. From the points A and B, *f g e*, draw perpendiculars to meet the horizontals drawn from the points similarly lettered in the elevation, and the intersections *e, f f, g g* will give the points through which the curves of the penetrations are to be drawn.*

Plate XXXIV.

Shows the projection of the objects when the plan has been rotated, so that the axis of the smaller cylinder is at an angle to the vertical plane. The lettering is omitted, but as all the lines of construction are shown, it is hoped that the student will be able to project the object with the aid of the instructions here given. It has repeatedly been shown that when an object is simply rotated on its axis, or on a solid angle, without altering the inclination, the *heights* of the various points will remain the same. This fact may be observed in a crane. When the weight has been raised as high as may be required, the crane is rotated, but the height of the top and of the weight will be exactly the same in which direction soever the crane may be turned, and thus the piece of ground overhung by the crane and weight will remain the same in form though altered in position. If, therefore, the plan and elevation given in Plate XXXIV. has been prepared, it will only be necessary to repeat the plan, placing the axis of the smaller cylinder at the required angle ; then perpendiculars raised from the various points in the plan may be intersected by horizontals drawn from the corresponding points in the elevation, and the inter-sections thus obtained will give the points required for the projection.

But, in practice, the whole of the object shown on Plate XXXIV. might be projected without referring to the previous one, and it is important that the student should understand this, as otherwise time would be lost. To project the object when at any angle, therefore, proceed in the following manner:—Draw the circle which repre-

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* The points x x x are not used in this projection, but will be sub-sequently referred to.
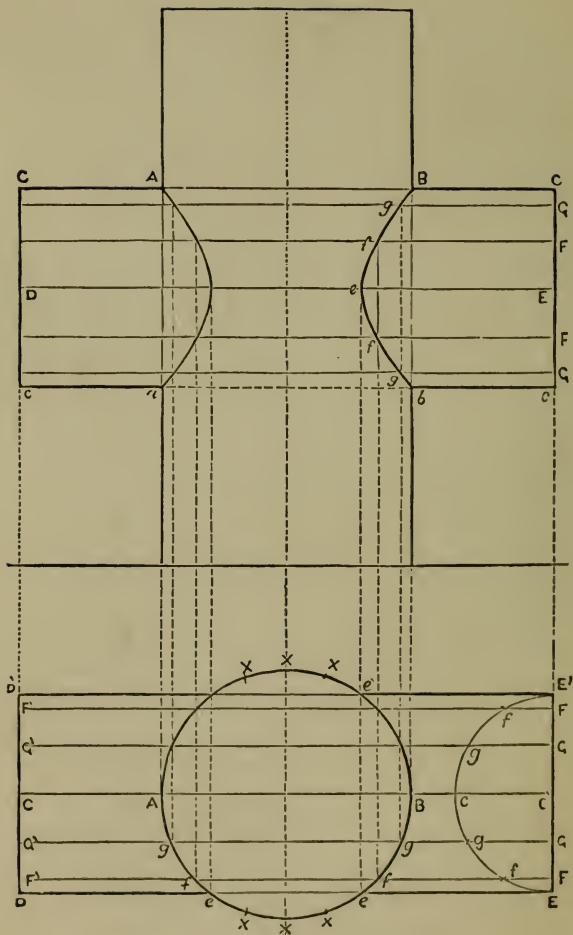
Plate XXXIII.
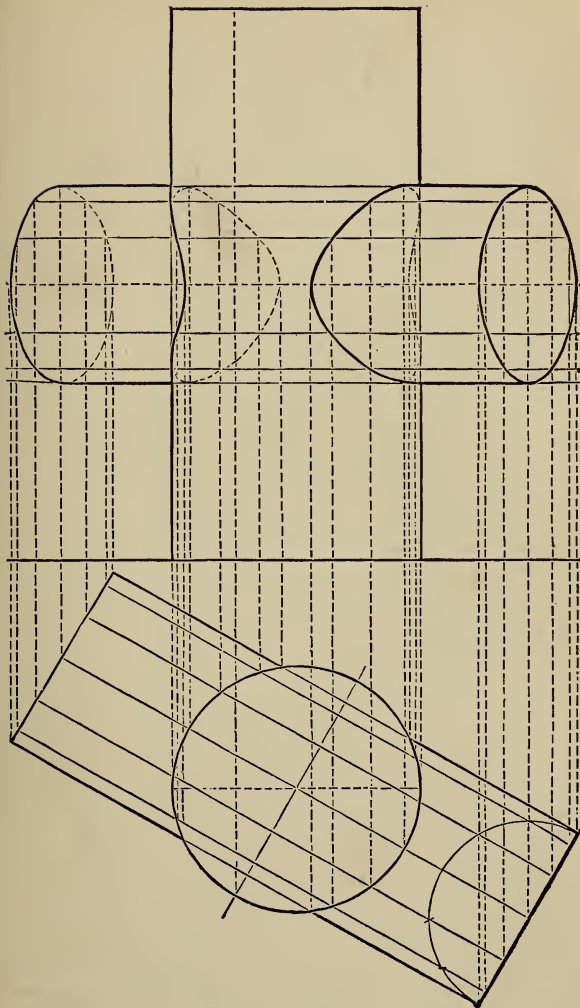
Plate XXXIV.

sents the plan of the larger cylinder. Draw a line through
the centre of this, making an angle with the intersecting
line corresponding to the angle which the axis of the
smaller cylinder is to make with the vertical plane. On
this line set off, on each side of the centre, half the length
of the smaller cylinder, and at these points draw lines at
right angles to the line of the axis. The plan of the object
will then be complete, and we proceed to project the
elevation from it. Draw a fine or dotted line through the
centre parallel to the vertical plane, and from the extremi-
ties of this diameter carry up the perpendiculars which
are to form the edges of the elevation of the larger
cylinder. Now it must be borne in mind that these are
*not the points from which the elevation of the cylinder
would be projected if the axis of the smaller one were
parallel to the vertical plane.* In that case the perpen-
diculars would be raised from the points where the axis
of the smaller cylinder cuts the circumference of the plan
of the larger one; but if this were done in the present
position of the plan, the elevation would be *narrower*
than the cylinder. All vertical sections of a cylinder are
parallelograms, and all those which pass through the
centre are equal. Still, reference to Plate III., Fig. 2,
will remind the student that the real size of a plane is
only obtained in the elevation when it is parallel to the
vertical plane; and it will be seen that the elevation of
the plane, of which the diameter of the plan, which is *at
an angle* with the vertical plane, is the elevation, would
not therefore be the projection of the largest section, and
would not represent the true width any more than the
elevation of the open door in Plate IV. represents its real
width, and it thus becomes necessary to draw the dotted
line referred to, so that the elevation may represent the
greatest width of the cylinder. Now draw another diameter
in the plan at right angles to the axis of the smaller cylinder,
and the extremities of this line will be the front and back
lines of the larger cylinder, which, if the axis of the smaller
one were parallel to the vertical plane, would be the centre
of the elevation; but as it has of course rotated with the
object, it is central no longer, but its relation to the heights
remains the same, however the larger cylinder may be
turned on its axis.

On this perpendicular, therefore, set off from the inter-

secting line the real height, and draw the horizontal line, which represents the top of the larger cylinder.

Mark on the perpendicular, too, the height at which the axis of the smaller cylinder intersects that of the larger, and draw a horizontal through the point.

Returning now to the plan, the preparation for the projection of the circular end of the smaller cylinder, as shown in Plate XXII., is necessary. On the line which forms the end in the plan draw a semicircle, and divide it into any number of equal parts. Through these points of division draw lines parallel to the axis of the smaller cylinder, which will be seen to pass through the plan of the larger one, and the intersections will be the plans of points " common to both " cylinders.

Now, from the points where these lines meet the straight line, which is the plan of the end of the smaller cylinder (on which the semicircle has been drawn), raise perpendiculars passing through the horizontal line which has been drawn across the elevation, and above and below this horizontal set off on the perpendiculars the *lengths of the lines drawn from the points in the plan from which they started* to the semicircle. Join the points thus obtained, and the projection of the end will be obtained. From each of the points through which the ellipse has been drawn now draw horizontal lines, and raise perpendiculars from the points in the opposite end of the plan ; the curve of the end of the smaller cylinder which is turned away must then be traced through these points, and it will be observed that, as only one side of the ellipse could really be seen in this position of the object, the other half is drawn in dots. It now remains to find the shape of the curve of penetration—that is, the curve generated where the smaller cylinder penetrates the larger, and this will be accomplished by finding the elevations of the points which in the plan were spoken of as " common to both " cylinders. From these points—that is, from the points where the lines drawn parallel to the axis of the smaller cylinder cut the circle, which is the plan of the larger one, erect perpendiculars cutting the horizontal lines in the elevation, which are in fact the elevations of the lines in the plan. The curves must then be traced through the intersections of these two sets of lines. The perpendiculars must

be drawn not only from the points on the front of the plan, but from those on the back part, and these cutting the horizontals will give the points through which the curve on the other side of the cylinder is to be drawn.

The reason why the perpendiculars at the back are to cut the same horizontals as those in the front, is that already pointed out—viz., that a point is not altered in height when the object on which it exists rotates on its axis in the manner shown in the diagram.

Plate XXXV.

It is now necessary to develop the larger cylinder, and to draw accurately upon the development the form of the aperture through which the smaller one shall pass. Now it must be borne in mind that this aperture, notwithstanding that it is to contain a cylinder, will not be a circle when the surface through which it is pierced is laid out flat.

This will be evident on referring to the plan in Plate XXXIII., where the length of the straight line e to e' is the *real* width of the penetrating cylinder. Whereas the distance between e and e', when measured on the circumference of the plan, would be much more, but as the axes of the two cylinders penetrate each other at right angles, the diameter in the elevation will remain unaltered.

The development of the general form of the cylinder will be accomplished by the method shown in Fig. 3, Plate XXIV.*

On this development draw a centre line A° representing A in the plan. The outer perpendiculars $B' B''$ will represent B in the plan. On each side of A° set off the lengths $g f e$, and erect perpendiculars; then the heights of the points correspondingly lettered in the elevation, marked off on these perpendiculars, will give points through which the development of the aperture may be traced.

It now only remains to develop the form of one of the ends of the penetrating or smaller cylinder. To do this, draw a horizontal line and erect a perpendicular E, and

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* The difference between this distance on the curve and on a straight line would be considerable, therefore divide it into several smaller parts, x x x, and set them off separately, by which means the difference will be lessened.
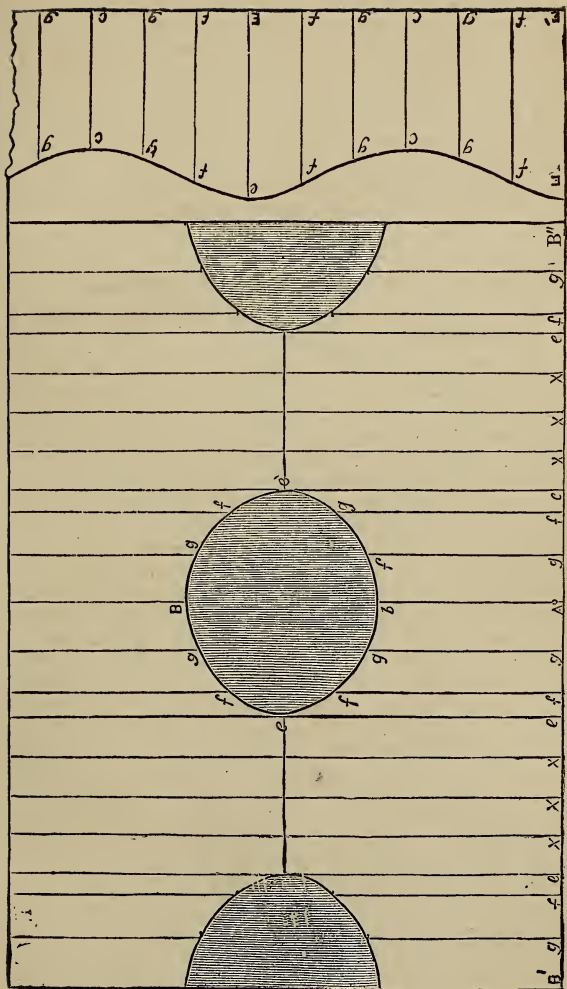
Plate XXXV.

on each side of this point set off the distances $f g$, $c g$, f E, into which the end of the smaller cylinder is divided, and from these points erect perpendiculars. On these set off the lengths of the lines between E E' and the plan of the larger cylinder—viz., E e, F f, G g, C' B, &c. The curve uniting the extremities of these perpendiculars will give the form in which the piece of metal is to be cut, so that when rolled and joined at its outer edges, it may form a part of a cylinder of the required size which will exactly fit to the aperture in the larger cylinder already explained.

Plate XXXVI.—To draw a Cone penetrated by a Cylinder, their axes being at right angles to each other.

Draw in the first place the mere elevation of the cone, A B C, and of the cylinder, D D' E E', intersecting each other in F F' G G'; and from these the general plan may be projected in the horizontal plane. The next problem for solution is the curve which will be generated by the intersection of the cylinder (which is a round body of *equal* diameter) with the cone (which is a round body of ever decreasing diameter). At D D' draw the perpendicular H I equal to the altitude of the cone, and from J, the middle line of the elevation of the cylinder, describe a semicircle equal to half the end of the cylinder. From I draw a line touching this semicircle in c, and reaching the intersecting line in C'. Between D and c and c and D' mark off any number of divisions, as b d. It must, of course, be understood that the greater the number of divisions marked off, the greater will be the number of points subsequently obtained, and of course, the greater the accuracy of the intersecting curve and development; but the object of the author is to make the operations as clear as possible, and therefore, in order to avoid one set of lines passing over another, and causing difficulties and confusion, he has only marked one division (b) in the upper and one (d) in the lower portion of the elevation. The student, who is expected to work this figure to a much

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NOTE.—To prove that I c' is a true tangent, draw c J, which will be at right angles to it. (See Tangents, in "Linear Drawing.")

Plate XXXVI.

G 2
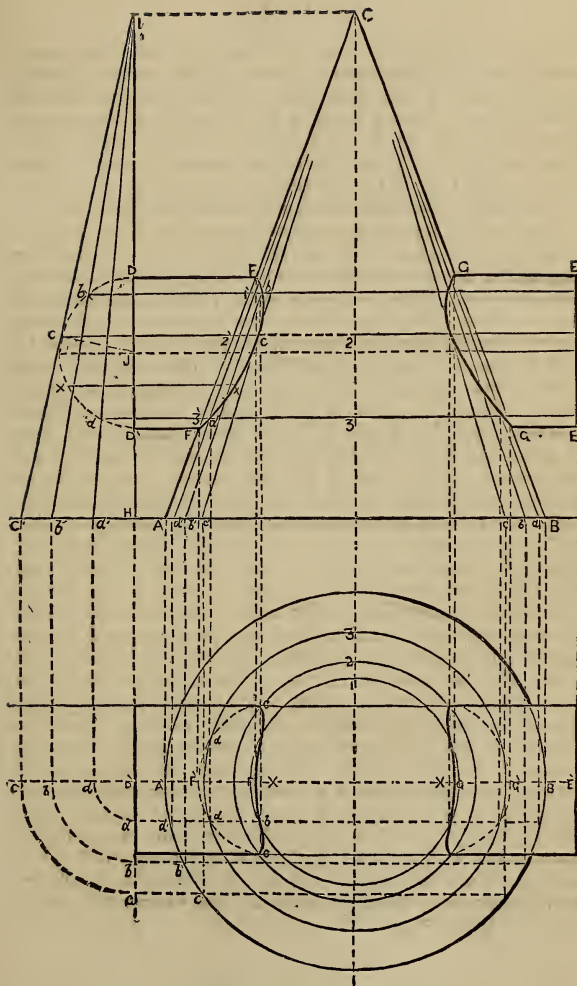
larger scale, will, however, do wisely to use many more points, all of which are worked in the same manner. From I draw a line through *b* cutting the intersecting line in *b'*, and from I draw a line through *d* cutting the intersecting line in *d'*. Through the centre of the *plan* draw the line X X and carry perpendiculars to it from *c' b' d';* and from D', with radius D *d*, D *b*, D *c*, draw arcs cutting I H produced in points similarly lettered.

From these points draw lines parallel to X X, cutting the plan of the cone in points to which (in order that the same line may be followed throughout) the same lettering is given—viz., *d' b' c'*. From these points carry perpendiculars cutting the base line of the elevation of the cone in *d' b' c'*, and draw lines from these points to the apex, C, of the cone. Intersect these lines by others drawn from *b c d* in the original semicircle, and through the points thus obtained the curve of penetration, starting at F and G, and ending in F and G', is to be drawn. It is now necessary to show on the plan the curve formed at the junction or penetration of the two bodies. Four points in these curves may at once be found by dropping perpendiculars from F F' and G G' in the elevation to cut X X in F F' and G G'. Now it will be remembered that every horizontal section of a right cone is a circle, and thus the lines parallel to the base on which the points *b c d* exist, are really edge elevations of circles, the diameter of which is regulated by their position on the cone. The length from point I on the edge of the cone to the axis, is thus the radius of the circle on which the point *b*, and the corresponding point beyond it, are placed. Therefore, with this radius describe a circle from the centre of the plan, and drop a perpendicular from *b*, cutting it in *b b*. Draw a circle from the same centre of the plan with radius 2 2', and a perpendicular from *c*, cutting it in *c c*. Draw a circle from the same centre with radius 3 3', and a perpendicular from *d*, cutting it in *d d*. Draw the curve F *d c b* F' *d c b*, which will be the plan of the aperture required. (Of course the corresponding lines on the other side will give a similar result.)

Plate XXXVII.—To Develop the Surface of this Cone.

The development of the simple surface of the cone having been fully demonstrated in Plate XXVIII., Fig. 2, it is not deemed necessary to repeat the process here; and we proceed, therefore, on the assumption that this simple development has been obtained, and requires the addition only of the shape of the aperture through which the cylinder is to pass. Draw the centre line, C A. On each side of A mark off the distances A' *d'*, A *b'*, A *c'*. from the plan, and mark the same from B and B. From all these points draw lines to C. On A and B B mark the heights A F' F from the elevation, and on the lines *d* C, *b* C, *c* C, in the development, mark the heights which the points *d*, *b*, and *c* are upon the lines similarly lettered in the elevation. These points joined will give the shape which the aperture must be cut on the flat material, so that when rolled, and the lines B B are brought together, the apertures shall be of the required shape and size.

To Develop one end of the Cylinder.

Set off on a straight line the lengths from D on the semi-circle, representing half of the cylinder (Plate XXXVI.) —viz., D *b*, *c*, X*, *d* D D'—and erect perpendiculars from them. Mark off on each of these perpendiculars the distance which each point correspondingly lettered is from the line D D', and unite these points by a curve drawn by hand.

The limits of this volume precluding any further illustrations of penetrations and developments, general principles only have been treated of. The subject will, however, be further enlarged upon in the future manuals, in which the studies will be worked out in a manner adapted to each special branch of industry.

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* The point x is introduced in order to divide the space between *c* and *d*, which would be too long to measure by one chord. (See note to Plate XXXV.)
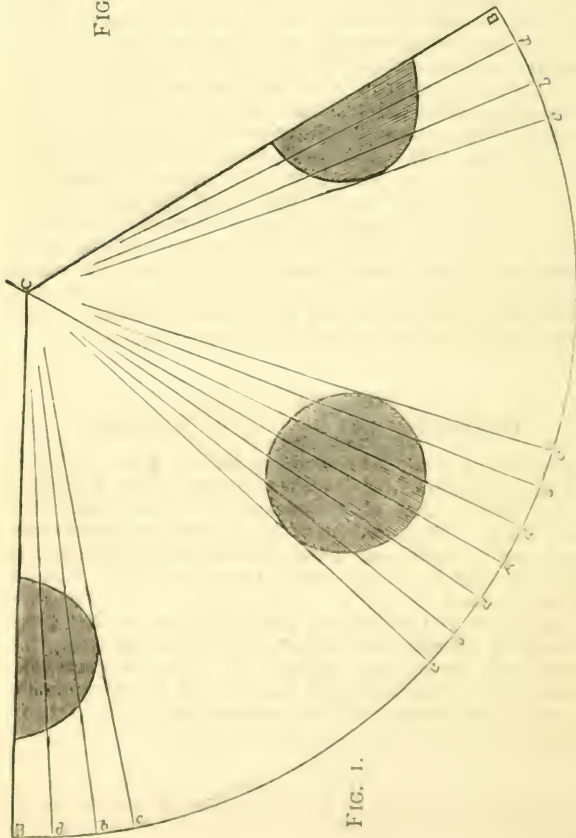
Plate XXXVII.
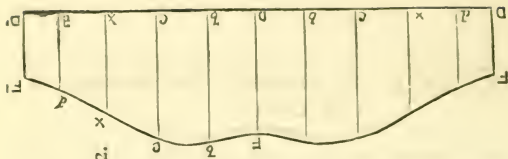
FIG. 1.

FIG. 2.

Plate XXXVIII.—The Helix.

If a piece of paper of the form of a right-angled triangle (A B C, Fig. 1) be rolled round a cylinder (Fig. 2), the
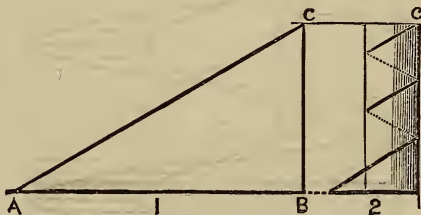
hypothenuse, or long side (A C), of the triangle, will generate a curve winding round the cylinder like a corkscrew. This is called the Helix, and it is this which forms the thread of a screw.

To describe a Helix.

Let the circle A G in Plate XXXVIII. be the plan of the cylinder around which the line is winding. Let the dotted perpendiculars A' and G represent the elevation, and let the distance A' to A 1 be the height which the curve has reached when it has travelled once round the cylinder, so as to be exactly over the point from which it started. This is called "one revolution," and is the "pitch," that is, the distance from thread to thread, in a screw. Divide the plan into any number of equal parts, as A, B, C, D, &c., and divide A' A 1 into the same number, viz., a, b, c, d, &c. Draw horizontals from a, b, c, d, &c., and perpendiculars from the corresponding points of the plan; then the intersections of B with b and C with c, &c., will give some of the required points. Now, it will be seen that the points H I J K L in the plan are immediately at the back of B C D E and F, and, therefore, the same perpendiculars will pass through them, and thus the inter-
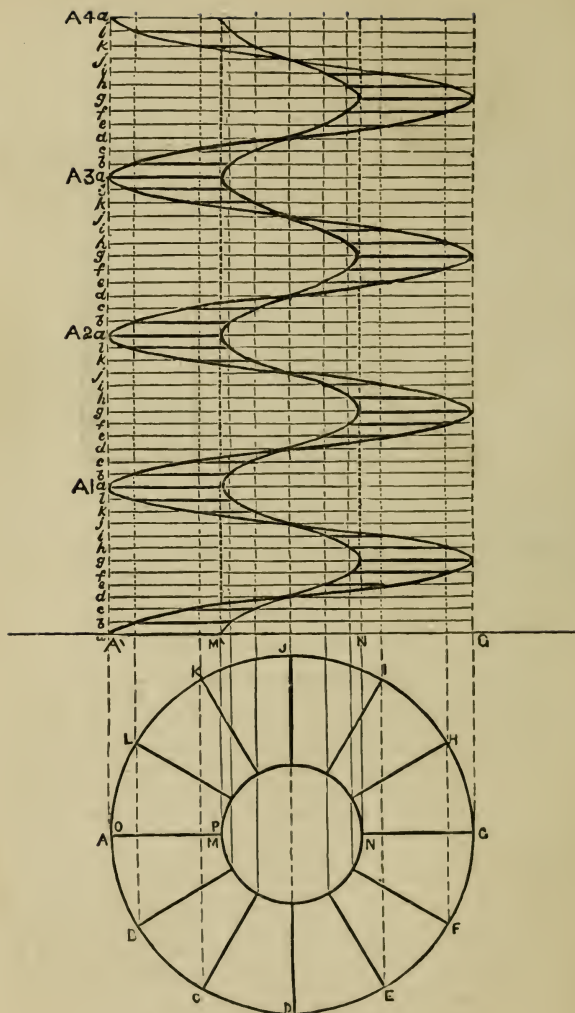
Plate XXXVIII.

sections of these lines with the horizontals correspondingly lettered will give the remaining points required for the formation of the curve. Through all the points now obtained the curve may be traced by hand. To continue the helix, repeat the height of the pitch as A 2, A 3, A 4; divide these spaces as before, and from the points draw horizontals to intersect the *perpendiculars* already drawn; for it will be evident that the corresponding points in each revolution will be immediately over each other.

Now let us suppose that, instead of a mere line being drawn round a cylinder, an inclined plane were to surround the smaller cylinder M N. You will understand this, perhaps, better if you cut out of paper the plan A G. Cut the smaller circle, M N, away altogether, and cut through the line A M. Place in the hole M N a cylindrical piece of wood of the exact size. Keep the edge A M fixed, but raise the edge O P. Then a rising plane, or walk, would be formed once round the small cylinder; and if this were constructed on a large scale, a person having travelled along this plane would have reached A 1, and be immediately over the point from which he ascended. *Now a staircase is only an inclined plane on which ledges, or stairs are placed to render the ascent easier.* For instance, let it be required to reach any height by means of an inclined plane. Of course this would be more readily accomplished by means of steps on the plane. And it will thus be seen, that if steps were placed on the inclined plane which surrounds the cylinder M N, the principle of a circular staircase would be developed. This principle will be further worked out in a subsequent volume. The points for the inner curve are obtained by perpendiculars taken from inner ends of the radii, cutting the corresponding perpendiculars.

Plate XXXIX.—To Project a Screw.

In the previous plate a flat surface was supposed to encircle a cylinder. Now if a "thread" of a triangular form (of which M N O is the section) were substituted, the result would be a V-threaded screw. To draw this, project the edge N of the thread as in the previous plate. P Q represents the plan of the smaller cylinder. The radii
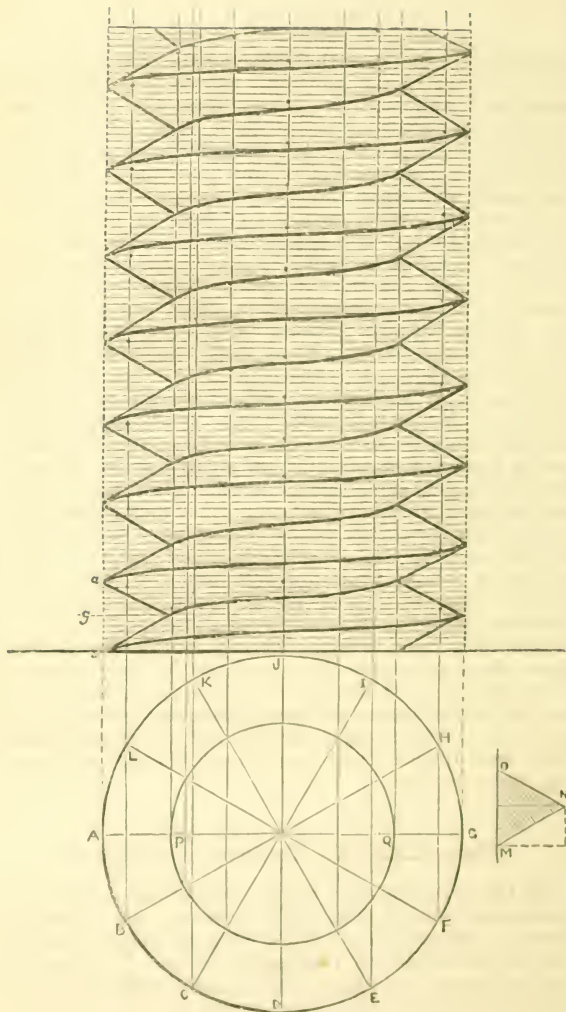
Plate XXXIX.

from the points in the outer circle will pass through this inner one; and from these intersections draw perpendiculars. Now the surface of the tooth is not, as in the last figure, a flat surface, but slanting, as will be seen by the lines O N and N M in the section; and therefore the curve for the inner helix will not start as before, from the same level as the outer one, but from the point where a perpendicular from P cuts the horizontal, *g*, midway between the edges of two teeth. The curve at the back is omitted here to avoid confusion; but the student, who is expected to work to at least twice the scale, is advised to follow the curve throughout, as the only means of ensuring correctness. The extreme points of the inner and outer curves being joined, will complete the projection of the screw. The whole subject of the projection of the various forms of screws will be fully considered in the volume devoted to mechanical drawing for engineers and machinists.

Plate XL.—To Project a small Church from the Plan.

The church, it will be seen, is made up entirely of simple solids—viz., square prisms of various lengths, triangular prisms, and a square pyramid; and as the student has already had some practice in these, he will find, it is believed, but little (if any) difficulty in following out the instructions, although the diagram is not lettered.

The building is to be considered in the first instance as formed of the square prisms only—that is, divested of the triangular prisms which form the roof, and also of the pyramid which forms the spire.

These solids, then, will be represented in the plan by two rectangles crossing each other at right angles, and as they are equal in width their intersection is a *square*, which is the plan of the tower; the shorter end of the longer rectangle then becomes the plan of the chancel, and the longer end the plan of the nave; the smaller rectangles form the plans of the transepts. It is advisable now to proceed with the projection of the body of the church from the plan. This operation is very simple, requiring only that perpendiculars should be drawn from
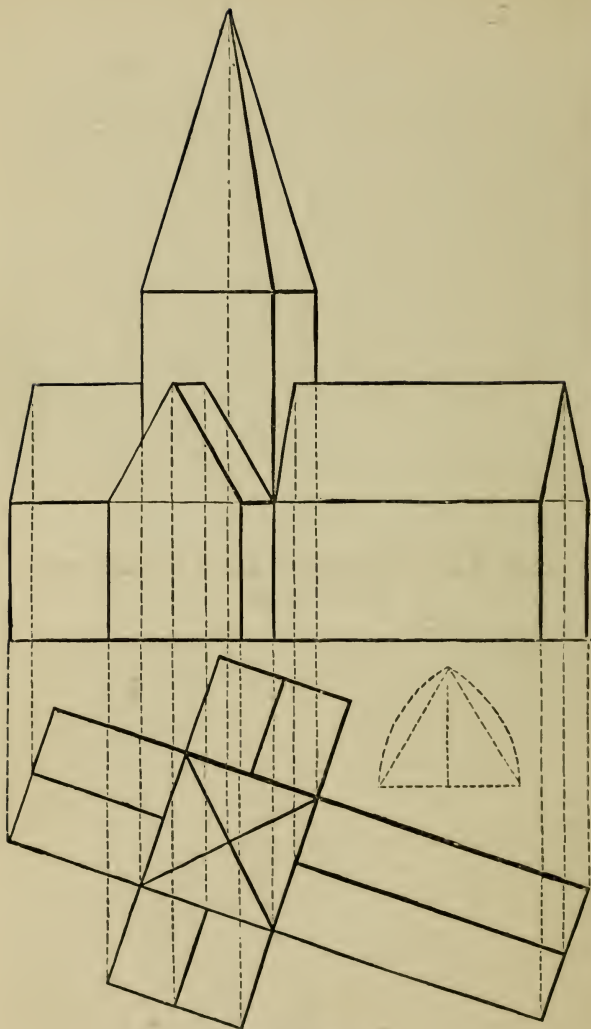
Plate XL.

the various points. From the two front angles of the transept which faces the spectator, therefore, draw perpendiculars, and a horizontal line cutting them off at a height above the intersecting line equal to the required height of the walls of the church. This horizontal line may be drawn of indefinite length, as it will regulate the height of the whole body of the building. A perpendicular drawn from the third angle of the transept (*i.e.*, the front left-hand corner of the square) will give the one edge of the tower of which the square is the plan, and a perpendicular drawn from the right-hand corner of the square will give, not only the side of the transept, but will, if continued, give the right-hand line of the front of the tower: further, a perpendicular raised from the distant right-hand corner of the square will give the side, the height of which may be determined by a horizontal to form the top line of the walls of the tower.

Next draw perpendiculars from the two angles of the right-hand end of the longer rectangle, and these carried up will give the projection of the rectangle, or wall forming the extreme end of the nave.

We now return to the plan, and draw the diagonals, which constitute the plan of the edges of the pyramidical spire. (See Plate XX., Fig. 2.) From their intersection draw a perpendicular, and on this mark the height of the required pyramid, this line being the axis. From the apex thus fixed draw lines to the upper angles of the projection of the tower, which will complete the spire.

Again reverting to the plan, draw lines through the middle of the rectangles, which will give the plans of the ridges of the roof. (See Plate XVI., Fig. 2.) From the point where the ridge-line meets the front of the transept draw a perpendicular, and mark on this, above the top line of the walls, the *perpendicular* height shown in the dotted triangle annexed. Join this point to the upper corners of the front of the transept, and this will complete its gable. From the apex of this triangle draw a horizontal line, and intersect it by a perpendicular drawn from the point where the ridge-line in the plan cuts the front line of the square. This intersection will give the point where the ridge meets the front of the tower. From this point draw a line parallel to that side of the triangle, and this will complete the visible transept ; the opposite

one is, of course, hidden by the body of the church, and could not therefore be seen in the present view. The student is, however, advised to project this object on the inclined plane, as shown in Plate XIII., when the upper portion at least of the hidden transept will be seen.

The rectangular part of the wall at the end of the nave has already been projected from the plan, and it now only remains to complete it by the addition of the gable.

It must be obvious that the gable-point will be immediately over the point where the ridge-line meets the end of the nave in the plan ; and, therefore, from this point erect a perpendicular, and carry it up between the two lines which represent the edges of the end of the nave. Draw a perpendicular, too, from the point where the ridge-line cuts the plan of the tower. A horizontal drawn from the gable-point of the transept will cut these perpendiculars, and give the corresponding point in the end of the nave, and in the part of the roof which meets the side of the tower. Produce this horizontal until it meets a perpendicular drawn from the end of the ridge of the chancel in the plan, and this will give the distant point in the ridge, and thus complete the projection of the church.*

An endeavour has been made to divest the subject of many difficulties ; for, although there is no " royal road " to learning, the road may be materially smoothened, and the traveller guided over obstacles which, unaided, might have been to him insurmountable barriers, each failure only weakening the power and the desire to make another attempt. Therefore in some cases the student has been left to complete a study, or to work out an exercise, unassisted by instr ctions, and a series of examination questions are appended, with the view of enabling him to test whether he has understood the lessons. It is hoped that this system will have had the effect of giving him confidence in himself ; and each success, however small, will be an encouragement to further effort. The great difficulty in the way of studying subjects of this class, is the aptness to *copy* the *diagrams,* instead of working out the *principles ;* and therefore the student is

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* For further projections from plans and elevations, see volumes on " Architectural Drawing " and " Building Construction."

urged to turn the plans and elevations in directions different from their position in the plates, and then to project them from other data. This will lead to *thinking;* and whoever has learnt to *think*, and has had that thinking properly directed, has only to add energy and perseverance to accomplish that success which should be the object of the ambition of every intellectual man.

ISOMETRICAL PROJECTION.*

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In all the previous constructions, it will have been observed that the projections have been obtained by the union of *plans* and *elevations*.

Isometrical Projection enables the draughtsman to work out views of buildings, &c., without these separate drawings, but still embodying both. This most useful system may be called the Perspective of the Workshop, as by its means we are enabled, not only to show in one drawing a view of the complete object, but all the lines of the projection may be measured by a uniform scale ; and hence the name, Isometrical, derived from two Greek words meaning " equal measures."

In this respect it differs from perspective, in which the sizes of all objects and lines diminish as they recede into the distance, according to distinct optical laws ; and it differs also from orthographic projection (which has formed the subject of our study hitherto), as in that branch of science the lengths of the lines are altered according to the angle at which the object may be placed. The whole system of isometrical projection is based on a cube resting on one of its solid angles, whilst its base is raised until the one solid diagonal—that is, the diagonal which connects the one angle of the top to the opposite angle of the bottom—is parallel to the horizontal plane. Then, if the cube be rotated on the angle on which it rests until the diagonal is at right angles to the vertical plane, the projection of the cube will be a regular hexagon. This will be clearly understood on referring to the following plate.

* Invented by Professor Farish, of Cambridge, about 1820.

Plate XLI.—The Isometrical Projection of a Cube.

Fig. 1 is the plan and Fig. 2 is the elevation of a cube, when raised on the solid angle *a*, so that the solid diagonal, A *b*, is horizontal, and thus when rotated on *a*, until A B is at right angles to the vertical plane, as in Fig. 3, the point *b* is hidden by the point A, and the projection will be seen to be a *regular hexagon*.

Now we know that when a regular hexagon stands on one angle, so that a line drawn from that angle to the centre may be quite upright, the two sides adjacent will be at 30° to the line on which the figure stands ; and this knowledge enables us to draw the isometrical projection of a cube without plan or elevation, but by means of the set-square of 30°, 60°, and 90°, by simply placing it with the long side of the right angle against the T-square (see Fig. 4), and having drawn one line of the hexagon, reversing the set-square and drawing the other, then, either moving the square along until its short edge is at the point of meeting of the two previously drawn lines, or turning it so that the short edge rests on the set-square, and thus drawing the vertical line. These three lines are then to be made equal, and the upper lines of the hexagon may be drawn, by again placing the set-square in the first and second position when the T-square is moved higher up on the board. All the lines forming the projection of the cube will thus be seen to be equal, but they will *not* be the real size which they would be in the plan or elevation, but will all of them bear the same proportion to the original measurement, and may therefore be measured by a uniform scale throughout.

To understand the construction of the isometrical scale, observe that the square, A B C D, Fig. 1, is represented in the projection, Fig. 3, by the lozenge, A' *b' c' d'*, and that all the other sides, which we know to be squares equal to A B C D, are represented by lozenges similar and equal to A' *b' c' d'*. In Fig. 5, therefore, this lozenge is placed within the square, and it will then be seen that the side D B of the square is at 45° to D E, whilst the side of the lozenge, D *b*, is at 30° to D E. The difference, then, between the triangle D E *b*, and the triangle D E B, is

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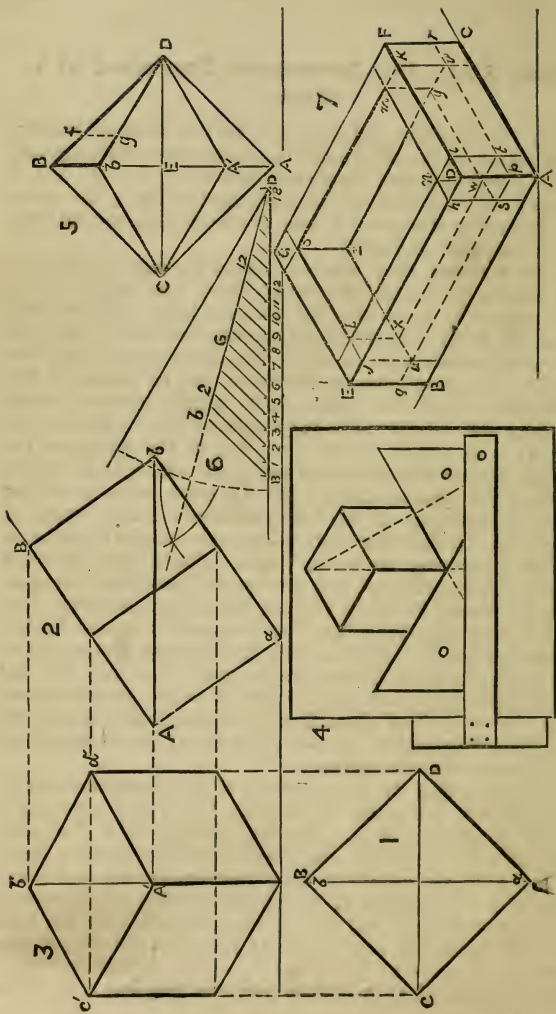
Plate XLI.

the triangle D *b* B, the angle *b* D B being 15°, and D B *b* being 45°.

It will therefore be plain that if a side of a cube be given, and we are required to find the side of the hexagon which should form the isometric projection of the cube, we need only take the given length as the base of a triangle, as D B. Construct an angle of 15° at one end (D) and of 45° at the other (B). Then the side D *b* of such triangle will be the required length of the side of the hexagon, and any divisions or parts marked on B D, as B *f*, may be transferred to *b* D, by drawing a line from *f*, parallel to B *b*, cutting *b* D in *g*; then *b g* will have the same proportion to B D that B *f* has to B D.

To Construct an Isometrical Scale.

Now let it be required to construct an isometrical scale, so that the object delineated may be one-twelfth of the real size. It will, of course, be understood, that this scale is *one inch* to the foot, as an inch is one-twelfth of a foot; and further, that if this inch be divided into twelve equal parts, each of the twelfths will represent the *inches* of the real measurement; that is, they will bear the same relation to an inch that an inch does to a foot—viz., one-twelfth; and, therefore, as in the proposed scale, an *inch* represents a *foot*, necessarily a twelfth of an inch represents an inch. The object to be projected is a box, 1′ 6″ long, 1′ 0″ wide, and 6″ high; the sides and bottom being 2″ thick.*

Draw the line B D, Fig. 6, an inch and a half long, representing the real length of the box—viz., a *foot* and a *half*, and mark on this the twelfths of inches, which are to *represent inches* on the scale. Draw at D a line at 15° to D B (which is most accurately done by drawing a line with your 30° set-square, and bisecting the angle). Draw at B a line at 45° to B D, cutting the line drawn from D in *b*; then the triangle B *b* D in Fig. 6 will be similar to the triangle D *b* B in Fig. 5, and therefore D *b* in Fig. 6 will have the same proportion to B D that the lines similarly lettered in Fig. 5 have to each other. From the points 1, 2, 3, 4, &c., in B D draw lines parallel to B

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* The student is reminded that one dash (′) over a figure means *feet*, and two dashes (″) *inches*; thus, 1′ 6″ is one foot six inches.

b, and these will divide *b* D proportionately to B D, and the divisions will thus, on the isometrical drawing, represent inches, and the line D *b* is an isometrical scale of $\frac{1}{12}$.

To Project a Box Isometrically.

We can now attempt the object, Fig. 7. By means of the set-square of 30°, draw the lines A B and A C; make A B 1′ 6″ long by the isometrical scale (the *line D b*), and make A C 1′ long. At A B and C draw perpendiculars.

Make A D 6″ high, and from D draw lines parallel to A B and A C, and cutting the perpendiculars B and C in E and F.

From E and F draw lines parallel to E F and D F, meeting in G, and this will complete the object as far as the mere block is concerned; and as a rule, it is advisable to project the general block view before attempting the detail.

From D, E, and F, mark off 2″ by scale—viz., *h i j k*, and from these draw lines parallel to D E F G, which intersecting in *l m n o*, will give the inner edge of the sides of the box, which, it will be remembered, are 2″ thick.

The bottom of the box is also 2″ thick, therefore on the perpendicular A set off A *p*, and draw *p q* and *p r* parallel to A B and A C.

From *h i j k* draw perpendiculars to cut these lines in *s t u v*, and from these points draw lines parallel to the sides of the box, cutting perpendiculars drawn from *l m n o* in *w x y z*, which will show the junction of the inner sides of the walls and the bottom, and will complete the projection.

Plate XLII.—To Project a Four-armed Cross.

Plate XLII. shows the isometrical projection of a four-armed cross standing on a square pedestal. Scale, $\frac{1}{4}$ of an inch to the foot; side of pedestal, 8 feet; height of ditto, 2 feet; complete height of cross, 14 feet.

The pedestal having been projected in a manner precisely similar to that by which the box in Plate XLI. was drawn, carry up the perpendiculars from the angles; make the perpendicular A B 14 feet high, and by drawing lines from B parallel to the sides of the base, complete
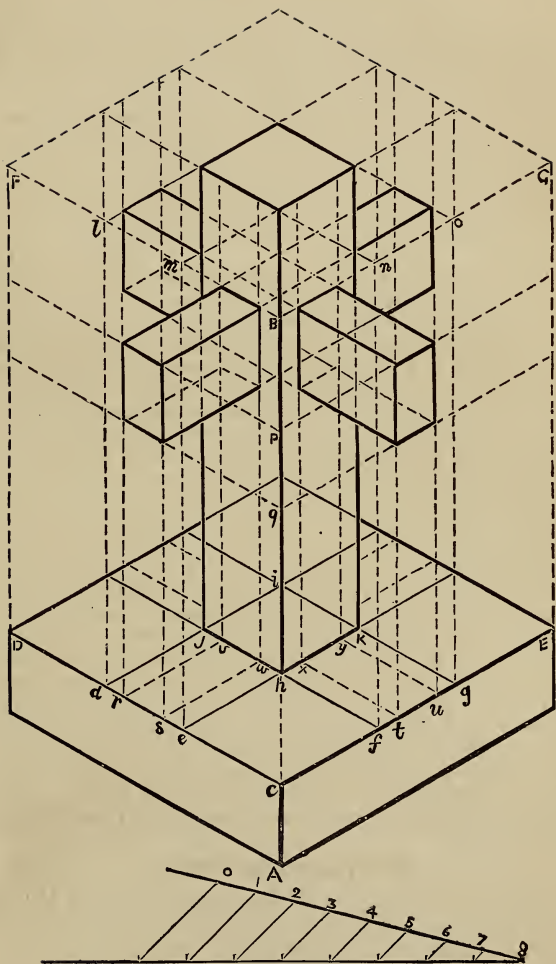
Plate XLII.

the top of a block which would contain the entire object; for, as the complete height of the cross is 14 feet, the top of the upright would be in the top of the block; and as the arms are 8 feet long from end to end, their extremities would be in the sides of the block, which may thus represent a glass case exactly containing the cross.

The thickness of the central upright is 2′ 0″; and as the width of the side of the pedestal is 8′ 0″, it follows that if 3′ 0″ be marked off from C to *e*, from D to *d*, from C to *f*, and from E to *g*, the spaces *d e* and *f g* will each be 2′ 0″.

From *d e* and *f g* draw lines parallel to the sides of the pedestal, which, crossing, will give the lozenge *h i j k*, which is the plan of the central upright. From *d e f g* draw perpendiculars to touch the edges of the top of the solid block, B F and B G in *l m n o*, and lines drawn from these points parallel to the sides will give the top of the central upright. On the front perpendicular A mark off *q* at 9′ 0″, and P at 11′ 0″ from the bottom, and from these points draw lines parallel to the sides C D and C E. These will give the heights of the top and bottom edges of the arms. But the arms are not so thick as the central upright, being only 1′ 0″; therefore between *d* and *e*, and *f* and *g*, mark off half a foot from each of the points. This will leave the spaces *r s* and *t u* each 1′ 0″ wide. From these draw perpendiculars, which, cutting the lines drawn from *p* and *q*, will give the ends of the arms; then draw lines parallel to the sides of the pedestal, cutting *h j* and *h k* in *v w* and *x y*, and from these points draw perpendiculars. From the angles of the ends of the arms draw lines parallel to the sides of the pedestal, cutting these perpendiculars, and these will complete the two arms which are turned towards the front. By producing these lines as shown in the diagram, the portions visible of the opposite arms may be drawn. All further detail will, it is hoped, be rendered clear by reference to the figure.

The Isometric Circle.

Projection does not deal with curves as such, but it becomes necessary to find points in rectilineal figures through which the curves pass, then to project the rectilineal figure, and trace the curve through the points thus
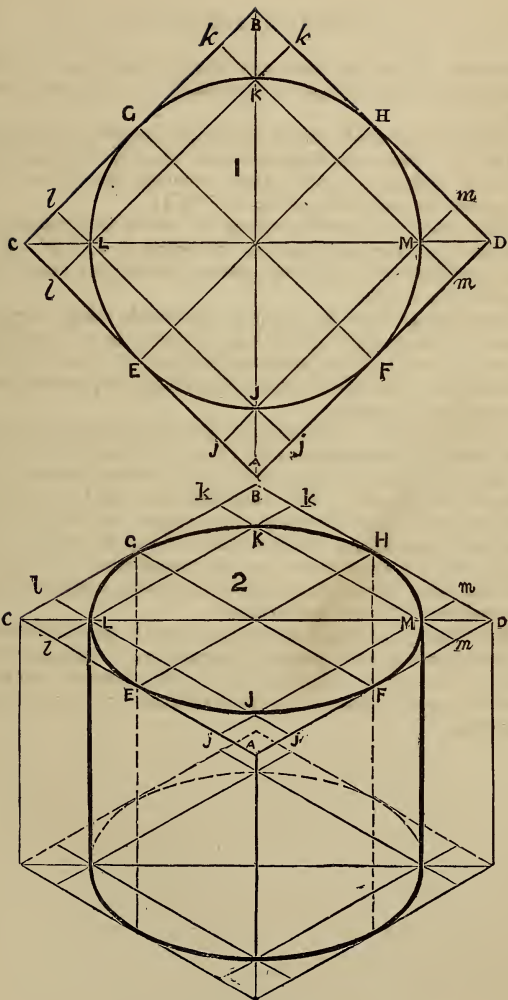
Plate XLIII.

obtained. Thus for isometrical purposes (as in radial perspective) the circle is enclosed in a square, as in Fig. 1.

Having drawn the circle, describe around it the square A B C D. Draw the diagonals, and also the two diameters, at right angles to each other, meeting the sides of the square in the tangent points E F G H.

The circle not only touches at these four points, but cuts through the diagonals in the points J K L M. Draw lines through each of these points, cutting the sides of the square in *j k l m*.

Proceeding now to project the circle thus prepared, draw the diagonal C D in Fig. 2, equal to C D in Fig. 1. From C and D draw lines at 30° to C D, intersecting in A and B. This will be the isometrical representation of the enclosing square.

The points E F G H and *j k l m* are obtained by marking from A the distances A *j*, *j* E, E *l*, and A *j*, *j* F, and F *m*, and drawing lines from these points parallel to the sides of the figure. The intersections J K L M will thus be obtained through which the ellipse, which is the isometrical projection of the circle, is to be drawn. The study may be carried on to the projection of a cylinder by repeating the operation for the bottom, and joining the intersections by perpendiculars.

The limits of this volume necessarily preclude further illustrations of this branch of projection. Various objects will, however, be delineated on this simple system in the volumes devoted to architectural and engineering drawing.

QUESTIONS FOR EXAMINATION.

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THE following questions are appended with the view of enabling the student to test his own knowledge, and as suggestions to teachers as to the mode of stating questions on this subject. It is hoped that the lessons given in this volume, and the application of them, will have shown the constructions upon which all the questions are based.

1. Give the plan and elevation of a line 3 inches long, when parallel to the vertical and horizontal plane, and 2 inches distant from each.

2. Give the plan and elevation of this line when it is at right angles to the vertical and parallel to the horizontal plane, its height being 2 inches from the ground.

3. Give the plan and elevation of the same line, when the former is a point, and the latter a vertical line 3 inches long.

4. Give the elevation* and plan of the same line when it is parallel to the vertical, but is inclined to the horizontal plane at 70°.

5. Give the plan and elevation of the line, when it is inclined at 70° to the horizontal, and 45° to the vertical plane.

6. A wire 3 inches long projects from a wall at 60° to the surface, and is parallel to the ground. Give the plan and elevation.

7. A plane 2″ × 3″ rests on its narrow edge in such a manner that its surface is at right angles to both planes. Give plan and elevation.

8. Give plan and elevation of the same plane, when its surface is vertical, but inclined to the vertical plane at 45°.

9. Give plan and elevation of the same plane when its shorter edges are at right angles to the vertical plane, and its surface inclined to the horizontal plane at 60°.

10. Give plan and elevation when the plane rests on one

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* When elevation occurs before "plan," it is to suggest that the elevation should be drawn firstly, and the plan projected from it, and *vice versâ*, the drawings being executed in the order of their names.

of its short edges, its surface being inclined at 60° to the horizontal plane, and its long edges being at 45° to the vertical plane.

11. A square plane of 3 inches side lies on the horizontal plane, its one diagonal being at right angles to the vertical plane, and the other parallel to it. Give plan and elevation.

12. Give elevation and plan when the plane rests on one of its angles, its surface being inclined at 40° to the horizontal plane, but its one diagonal remaining at 90° to the vertical plane.

13. Give plan and elevation of the same plane when one of its diagonals is at 45° to the horizontal, and 60° to the vertical plane, the other diagonal being parallel to the horizontal plane.

14. A cube of 2 inches side stands on the horizontal plane, with two of its faces parallel to the vertical plane. Give its plan and elevation.

15. Draw its plan and elevation when standing on one of its sides, the opposite one being horizontal, and the others being at 45° to the vertical plane.

16. Give plan and elevation when resting on one of its solid angles, one diagonal of the base being at 50° to the horizontal, and the other at 90° to the vertical plane.

17. Draw elevation and plan of the same cube, when resting on one of its *edges*, so that two of its sides are vertical and the rest make angles of 45° with the horizontal, but are at right angles to the vertical plane.

18. Add the shape (the *development*) of the piece of metal or other substance which on being folded would form the above-named cube.

19. There is a stick of timber 2 inches square at base, and 5 inches high. Give the true shape of a section caused by a plane entering at one angle of the top, and emerging at the opposite angle of the base.

20. Give the development of one portion of this square prism.

21. Give plan and elevation of a triangular prism when resting on one of its long faces, the surface of the triangular end being at 50° to the vertical plane.— The end is an equilateral triangle of 2 inch edge, and the length of the prism is 3½ inches.

22. Give plan and elevation of the same prism when the edge of the end on which it rests is at 50° to the

vertical plane, and the under side is inclined to the horizontal plane at 35°.

23. Add the development of this prism.

24. Draw the plan and elevation of a regular pentagon of 1 inch side when resting on one of its angles, so that its surface is at right angles to the vertical, and at 60° to the horizontal plane.

25. Give the projection of this polygon when the line joining the angle on which it rests to the middle of the opposite side is at 40° to the vertical plane, the inclination to the horizontal plane remaining the same as in the last figure.

26. There is a hexagonal prism of 1 inch side and 4 inches long. Draw plan and elevation when standing on its end, with two of its faces parallel to the vertical plane

27. Give the plan and elevation of the same prism, when the axis is vertical and one of its faces is at 40° to the vertical plane.

28. Give elevation and plan of the same prism when two of its faces are parallel to the vertical plane, and the prism is so inclined that the axis is at 50° to the horizontal plane.

29. Draw the plan and elevation when the prism rests on one of the solid angles, and the axis is at 50° to the horizontal and 45° to the vertical plane.

30. Project the prism when lying on one of its long faces, the axis being at 40° to the vertical plane.

31. Give the true section caused by a plane passing from one angle of the top to the opposite angle of the bottom.

32. Draw the development of the prism, marking on it the line of section, as per last figure.

33. Give the plan and elevation of a pyramid formed of four equilateral triangles of 3 inches side, when one edge of the base is at 35° to the vertical plane.

34. Draw the plan and elevation of a pyramid, the base of which is a square of 2 inches side, and the altitude of which is 3 inches, when its axis is vertical, and two of the edges of the base are parallel to the vertical plane.

35. Draw the same pyramid when the edges of the base are at 45° to the vertical, and the axis perpendicular to the horizontal plane.

36. Give the projection when the pyramid rests on one angle of the base, the surface of which is inclined at 30° to the horizontal plane.

37. Draw the true shape of a section caused by a plane passing from a point in one of the long edges at 2 inches, to a point in the opposite edge 1 inch from the bottom. Give the development of the pyramid, marking on it the line of section.

38. Give plan and elevation of a hexagonal pyramid when two of the edges of the base (1 inch long) are at 20° to the vertical plane, the altitude being 2½ inches.

39. Draw elevation and plan of this pyramid when lying on one of its triangular faces, with its axis parallel to the vertical plane.

40. A circular disc (1½ radius) stands so that one diameter is vertical, and another at right angles to the first is at 50° to the vertical plane. Give plan and elevation.

41. Give elevation and plan of the same circular disc, when resting on the end of one diameter, which is parallel to the vertical plane, the surface being at 40° to the horizontal plane.

42. Draw the plan and elevation of the same disc, when the diameter is at 40° to the horizontal and 60° to the vertical plane.

43. A circular slab of stone, such as a mill-stone, 4 feet diameter and 1 foot high (*to be represented by inches for feet*), lies on the horizontal plane. Give the plan and elevation.

44. A second circular slab, 3 feet diameter and 1 foot high, rests on a slab, similar to the last; their centres being coincident. Draw the plan and elevation.

45. Draw the elevation and plan of these two slabs, one placed on the other, as above, when their circular surfaces are inclined at 40° to the horizontal plane.

46. A cylinder, 4 inches long and 2 inches diameter, stands on its circular end. Give the plan and elevation.

47. Draw the plan and elevation of the same cylinder when lying on the horizontal plane, its axis being parallel to both planes of projection.

48. Give plan and elevation of the cylinder when lying on the horizontal plane, its axis being at 60° to the vertical plane.

49. Draw the plan and elevation of a cylinder 4 inches

long and 2 inches diameter, when the axis is inclined at 60° to the horizontal and 45° to the vertical plane.

50. Give the true section caused by a plane passing through the middle point of the axis at 45° to it.

51. Draw the development of this cylinder, marking on it the line of section.

52. A cylindrical pipe, of 2 inches diameter, is to be cut so as to turn a right angle. Give plan and elevation, showing the section-line.

53. Give the elevation and plan of one of the parts when resting on the sectional surface.

54. Give the true shape of the section, and the development, showing how both parts of the elbow may be cut out of the same piece of metal without any waste.

55. From piping of the same diameter, construct a double elbow-joint, one end of which bends one way and the other the opposite. Give development of the three parts to be cut out of one piece without waste.

56. The same piping is to be carried round three sides of a square room (size at pleasure). Give development, showing the section-line.

57. A pipe of sheet iron (2 inches diameter) is to be joined so as to turn an angle of 120°. Show on an elevation the inclination of the line of section, and show on a development the line in which the metal must be cut to form the required parts without any waste.

58. Given a cone of 2½ inches base and 3½ inches altitude. Draw the plan and elevation of this cone when standing on its base.

59. Give elevation and plan, when the cone lies on the horizontal plane, its axis being parallel to the vertical plane.

60. Draw the projection of the cone, when lying on the horizontal, with its axis at 45° to the vertical plane.

61. Project the cone when resting on one end of the diameter of the base, the axis being inclined at 70° to the horizontal plane.

62. Project the cone, when the axis is inclined at 70° to the horizontal and 45° to the vertical plane.

63. Draw the true section of the same cone caused by a plane at 40° to the surface of the base, which enters at ¼ inch from the bottom. Add the development, marking on it the line of section.

64. Draw the parabola resulting from a plane entering the base of a similar cone at $\frac{3}{4}$ inch from the centre.

65. Draw the hyperbola resulting from a section-plane entering the base of a similar cone at $\frac{3}{4}$ inch from the axis.

66. A pipe 2 inches square is penetrated by another of 1 inch side. The smaller one passes through 2 sides of the larger, their axes being at right angles to each other. Give elevation and plan when two faces of each of the pipes are parallel to the vertical plane.

67. Project this object when the two faces, which in the last case were parallel to the vertical plane, are at 60° to it.

68. Give the development of the larger pipe, showing the exact shape of the aperture through which the smaller one is to pass.

69. Give the elevation and plan of the object when the smaller pipe penetrates the sides of the larger at 60°.

70. Draw the development of the larger pipe, showing the apertures, and of one piece of the smaller one.

71. A square pipe of 2 inches side is penetrated by another of $1\frac{1}{2}$ inch side, their axes being at 60° to each other, and parallel to the vertical plane ; and two *edges* of the smaller meeting two edges of the larger pipe. Give the elevation and plan.

72. Draw the plan and elevation, when two faces of the larger pipe are parallel to the vertical plane.

73. Draw the development of the larger pipe, showing the shape of the apertures through which the smaller one is to pass, and also one of the ends of the smaller pipe.

74. A cube of 3 inches side stands on the horizontal plane, and is surmounted by a square pyramid, formed of a base and four equilateral triangles of $3\frac{1}{2}$ inch side. Give elevation and plan, when two faces of the cube and two of the sides of the base of the pyramid are parallel to the vertical plane.

75. Draw the elevation and plan of this object, when the faces are parallel to the vertical plane, as in the last question, but when the base is inclined at 25° to the horizontal plane.

76. Draw the plan and elevation of the object, when the sides of the cube are at 50° and 40° to the vertical plane.

77. Give plan and elevation of the object, when the faces of the cube are at 45°, and two of the sides of the

base of the pyramid are parallel to the vertical plane, their axes being coincident.

78. Draw the shape of the piece of metal to form a gas-shade, 20 inches wide across the circular base, 6 inches across the top, and 10 inches perpendicular height. (To be worked $\frac{1}{2}$ size.)

79. A cylindrical coal-scuttle is to be made of sheet iron; it is to be 10 inches in diameter and 18 inches high at the highest part, the lid to be inclined at 45°. Draw the shape the metal is to be cut to form this object, and the exact shape of the lid. (To be worked $\frac{1}{2}$ size.)

80. A cylinder, $2\frac{1}{2}$ inches diameter and 6 inches long, is penetrated by another of $1\frac{1}{2}$ inch diameter and 5 inches long, their axes being at right angles to each other, and intersecting at their centres. Show the mode of obtaining the curves of penetration. Develop the larger cylinder and one of the ends of the smaller one.

81. Draw the plan and elevation of this object when the axis of the larger is parallel and of the smaller at 60° to the vertical plane.

82. There is a solid cross, formed by a central cube of 1 inch side, on each face of which another similar cube is fixed. Give the plan and elevation when two vertical faces of the original cube are parallel to the vertical plane.

83. Draw the plan and elevation when of the two adjacent vertical sides of the original cube one is at 60° and the other at 30° to the vertical plane.

84. Project this object when resting on one angle of the base of the lowest cube, which is inclined at 30° to the horizontal plane, the diagonal being parallel to the vertical plane.

85. A cone, the base of which is 4 inches and the altitude of which is 5 inches, is penetrated by a cylinder of 2 inches diameter. The axis of the cylinder intersects that of the cone at right angles, at 1 inch from the ground. Draw the plan and elevation when the axis of the cylinder is parallel to the vertical plane.

86. Project this object when the axis of the cylinder is at 60° to the vertical plane.

87. Draw the development of the cone (showing the apertures of penetration) and of one end of the cylinder.

88. A thread winds round a cylinder 6 inches high, and of 2 inches diameter, and reaches the top in 6 revolutions. Project the helix curve thus generated.

89. Project a V-threaded screw of 3 inches diameter and $\frac{1}{2}$ inch pitch.

90. There is a solid formed of two equal square pyramids of 2 inches base and 3 inches altitude, which are united by their bases. Draw the elevation and plan when the object rests on one of the triangular faces of one of the pyramids, the axis of the object being parallel to the vertical plane.

91. Give the projection of the object, when resting on one of the faces of one of the pyramids. The axis is at 45° to the vertical plane.

92. Draw the elevation and plan when the object rests on an *edge* of one of the pyramids, the axis being at 60° to the vertical plane.

93. Construct an isometrical scale of $\frac{1}{10}$ of an inch to the foot. Show 20 feet.

94. Draw an isometrical projection of a plane square of 2 inches side.

95. Give an isometrical projection of a pavement consisting of squares of 1 foot side. Scale, $\frac{1}{2}$ inch. Show 5 squares in width and 12 in length. Tint the alternate squares in pale Indian ink.

96. Draw an isometrical projection of a cube of 2 inches edge.

97. Draw the isometrical projection of a box 3 feet square and 2 feet high, made of wood 3 inches thick. Scale, 1 inch to the foot.

98. There is a block of stone, 6 feet square and 1 foot high; on this rests another, of the same height and 4 feet square; and on this again a third block, of the same height and 2 feet square, is placed, the centres of the three blocks being over each other. Give the isometrical view of the group. Scale, $\frac{1}{2}$ inch to the foot.

99. A cylinder of 2 inches diameter and 4 inches long lies so that its end is vertical. Give the isometrical projection.

100. There is a stool the top of which is a square of 12 inches side, the height 18 inches, and the thickness of the legs 2 inches (the other measurements at pleasure). Scale, 2 inches to a foot. Draw an isometrical view of this object.

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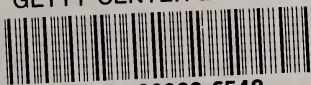
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